

# Uncertainty and Misallocation: Theory and Industry Evidence

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April 11, 2018

## Abstract

In a canonical model of partial investment irreversibility, real options considerations imply that firms only actively invest or disinvest when the mismatch between their productivity and their capital stock is large. The calibrated model indicates that uncertainty shocks are particularly detrimental to growth in industries with high capital depreciation rates. A differences-in-differences regression using industry growth data from a large sample of countries is consistent with these findings.

*Keywords:* Uncertainty, irreversibilities, depreciation, investment lumpiness, real options, misallocation.

JEL Codes: D80, E22, E32.

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# 1 Introduction

An extensive literature studies the impact of economic uncertainty on aggregate activity. There is a general consensus that uncertainty tends to lead to declines in aggregate economic activity. This applies even when economic uncertainty is measured net of any shocks to levels of fundamentals that might coincide with increases in uncertainty – see Baker and Bloom (2013).

However, there is less consensus regarding the key channels through which this occurs. Examples include the real options channel, which relies on investment irreversibility to induce caution in investment decisions when uncertainty is high, and the risk aversion channel, which leads firms to experience higher borrowing costs in uncertain times.<sup>1</sup>

This paper tests the real options channel by looking at *industry variation*. We argue that a key implication of the irreversibility underlying the real options channel is that they do not affect industries symmetrically. In particular, we identify a new way in which real options considerations and the delays in investment they induce can interact with uncertainty: the increased misallocation of resources induced by caution in investment decisions. We show that, in the empirically relevant range of depreciation rates, this channel will be most deleterious to growth in industries where depreciation is relatively rapid.

We develop a parsimonious model of industry dynamics in the presence of investment irreversibilities. Firms have opportunities to grow or shrink based on their idiosyncratic productivity, which varies according to a Markov process as in Hopenhayn (1992). Investment irreversibilities have two effects on investment patterns that are related to uncertainty. First, they introduce a motive of caution. Firms will not wish to actively invest nor disinvest, since any current or future disinvestment is imperfectly reversible, unless there is a significant mismatch between their productivity and their existing capital stock. Second, when firms do invest, it is because this mismatch is large, so they are more likely to invest in "lumps." Lumpy investment is a feature of investment dynamics known at least since Doms and Dunne (1998).

Into this environment we introduce uncertainty shocks, defined as periods when the productivity process is a mean-preserving spread of what is observed in normal times.<sup>2</sup> We argue that irreversibilities lead uncertainty and capital depreciation to interact in important ways. The key is that rapid depreciation leads mismatches between the firm's productivity

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<sup>1</sup>See Dixit and Pindyck (1994) and Gilchrist et al (2014). See also Bloom (2014) for a recent survey.

<sup>2</sup>The model can be interpreted in terms of common productivity shocks, purely idiosyncratic productivity shocks or any combination thereof.

and the firm's capital stock to develop *more rapidly*. When depreciation is low, mismatch occurs mainly due to productivity change: if productivity does not change or changes little, the capital stock remains similar to what it was before. When depreciation is high, mismatch occurs rapidly even when productivity does not change much. In times of high uncertainty, with both capital depreciating rapidly and productivity being unstable, mismatch is more frequent and labor productivity declines as a result of increased misallocation. This also means that investment lumps are more frequent in industries with rapid depreciation, as firms who do not experience any significant change in productivity nonetheless find themselves having to reinvest to maintain a reasonable scale of operations. Consequently, industries where capital depreciation is higher will likely grow disproportionately slowly in times of high uncertainty. We calibrate the model economy and find that it is consistent with all these hypotheses. In particular we find that uncertainty shocks particularly depress labor productivity growth (as well as output) in industries with rapid depreciation, and that firms in these industries experience more frequent investment lumps, consistent with the "misallocation" hypothesis.

Interestingly we also find that, as depreciation approaches 100 percent, the misallocation effects disappear and uncertainty particularly *favors* growth, as in Oi (1966), Hartman (1972) and Abel (1983). However, this is because such rapid depreciation makes irreversibility costs irrelevant: the "bounds of inaction" in investment disappear and there is less room for misallocation. In the calibrated model we find that these effects are only important for depreciation rates outside the empirically relevant range.

Finally, we test these key predictions – that depreciation and lumpiness are correlated, and that high-depreciation industries and high-lumpiness industries should grow disproportionately slowly in times of high uncertainty – using data on manufacturing industries and uncertainty shocks from a large number of countries.<sup>3</sup> We show these results are robust to a variety of controls, and to allowing for other industry characteristics that might be supportive of alternative mechanisms for uncertainty to affect growth. We conclude that real options mechanisms, resulting from partial investment irreversibilities, are key for understanding the response of growth to uncertainty shocks. We also conclude that a canonical model of investment irreversibilities captures all these features.

This paper contributes to the long-run debate about the main sources and propagation mechanisms of uncertainty. Real options theory shows that when investment is irreversible,

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<sup>3</sup>We focus on manufacturing industry growth data because of the difficulty of identifying large cross-country data sets with service sector data: naturally a study using a broader set of industries would be a useful extension.

greater uncertainty induces caution among firms, slowing reallocation of resources and lowering aggregate productivity. In contrast, risk aversion theory argues that when firms are risk averse, greater uncertainty (including a greater risk of default) may lower economic activity by increasing the cost of external funds. See for example Gilchrist et al (2014) and Alfaro et al (2016). Our results identify the real options channel as the key mechanism through which uncertainty leads to declines in economic activity. Specifically, we identify the role of irreversibility through the depreciation rate and investment lumpiness.

This study also relates to the capital misallocation literature, for example Hsieh and Klenow (2009) and Bartelsman et al. (2013). Eisfeldt and Rampini (2006) emphasize the importance of capital misallocation for understanding business cycles. In contrast, we discuss the increases in capital misallocation due to wait-and-see effects in response to *uncertainty shocks* (as in Bloom (2009)), and argue that the source of such misallocation is linked to rapid capital depreciation. We show that there exists a "range of inaction" even though firms face capital misallocation during uncertainty, and that the impact on misallocation of this range is related to depreciation rates.

Our findings also contribute to the long literature on real options and flexibility in manufacturing, including Kulatilaka and Marks (1988), McDonald and Siegel (1985,1986) and Dixit and Pindyck (1994). Flexibility in manufacturing can be interpreted as machine flexibility, material handling system flexibility or operational flexibility. Our model provides a parsimonious model of the irreversibilities which cause inflexibility in firm's investment decisions, as well as providing new evidence to the importance of flexibility in the manufacturing sector.

Finally, this paper contributes to the literature on industry dynamics over the business cycle. Braun and Larraín (2005) find that industries dependent on external finance grow disproportionately slowly in business cycles. Samaniego and Sun (2016) study a broad set of industry characteristics and show that growth in labor intensive industries is especially sensitive to contractions. This paper focuses on industry depreciation rate and lumpiness, and adds value to the literature by revealing more dimensions of industry technological features underlying aggregate fluctuations.

Section 2 describes the model economy. Section 3 provides a quantitative analysis of the model economy and derives empirical predictions. Section 4 outlines the estimation strategy and Section 5 describes the data to be used. Section 6 delivers the empirical results and robustness checks. Section 7 delivers concluding remarks.

## 2 Model Economy

We begin with a canonical model of firm dynamics with investment irreversibilities. It builds on the model of Hopenhayn (1992), where firms experience productivity shocks that affect their optimal input use, and adds partial investment irreversibilities. Time is discrete and there is a continuum of firms in the industry.

The environment is subject to uncertainty shocks. Variable  $v_t \in \{0, 1\}$  is a volatility variable, where  $v_t = 1$  is referred to as an *uncertainty shock*. The volatility process evolves according to  $v_{t+1} \sim F_v(v_{t+1}|v_t)$ . In what follows we will define an environment where firms experience idiosyncratic shocks that depend on the level of volatility, yet volatility itself has no effect on the level of fundamentals – in this case, expected productivity. Thus, there are no aggregate shocks other than volatility itself.

Firms are subject to idiosyncratic productivity shocks. The volatility variable  $v$  which affects the evolution of idiosyncratic productivity shocks. Denote a firm's productivity in period  $t$  as  $z_t \in Z \subseteq \mathbb{R}$ , where  $z_{t+1} \sim F_z(z_{t+1}|z_t, v_{t+1})$ . Assume  $F_z(z_{t+1}|z_t, 1)$  is a mean-preserving spread of  $F_z(z_{t+1}|z_t, 0)$  for all  $z_t \in Z$ . Notice this implies that *expected* productivity does not depend on the realization of  $v_t$ . We will maintain all other aspects of the environment constant so that uncertainty is characterized solely as a mean-preserving spread of productivity shocks: there are no shocks to levels of economic fundamentals nor other level variables in the system. Any impact on levels of economic activity will be due to changes in the optimal investment policies of firms.

Firms own their own capital, which they may purchase at price  $p_k$  and which depreciates at rate  $\delta$ . New capital is created through investment of a final good as in a typical growth model, so we set the final good as the numeraire and set  $p_k = 1$  accordingly. In order to *remove* capital, however, a share  $\kappa \in (0, 1)$  of it is destroyed – so its resale price is only  $1 - \kappa$ . Parameter  $\kappa$  represents the extent of partial investment irreversibility. The presence of  $\kappa$  can be interpreted as being due to capital being customized upon installation – so that it does not transfer with full functionality when sold to other firms – or because capital is damaged in the removal process.

Each period a number of entrants  $e$  is born, drawing their initial productivity  $z_t$  from a distribution  $\psi(z_t)$ . A given firm closes at the end of each period with probability  $\lambda(z_t)$ .<sup>4</sup>

Agents in this environment discount the future at rate  $i$ .

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<sup>4</sup>We assume that when the firm shuts down its value is zero: in a different context Samaniego (2006b) argues this will likely be the case in practice as the owners' stake in a failing firm will be eaten up by other claims on the firm's assets.

The firm produces a good at price  $p$  which is set to one without loss of generality since it only has a level effect. It pays a wage  $w$  for each unit of labor it hires.

**Remark 1** *Notice that we have set prices to be constant over time. An equivalent assumption would be to make the pricing process part of the productivity process.<sup>5</sup> One interpretation of our environment is that it describes a small open economy where prices are largely given by conditions in international markets, and where wages are rigid at a cyclical frequency as in for example Shimer (2012). Regardless, the main assumption we wish to ensure is that uncertainty shocks have no level effects, so as to isolate the impact of uncertainty on behavior net of any level effects. As mentioned, the empirical literature makes great effort to identify separately the impact of uncertainty shocks and of level shocks that might coincide with them, see for example Baker and Bloom (2013).*

Given the stationarity of the economic environment, it lends itself to analysis using recursive optimization techniques. The firm's value function is

$$V(z, k_{-1}; v) = \max_{n, k} \left\{ zk^\alpha n^\theta - wn - (k - k_{-1}) - \kappa \max\{0, k_{-1} - k\} + \frac{1 - \lambda(z)}{1 + i} EV(z', k(1 - \delta); v') \right\} \quad (1)$$

where  $k_{-1} = 0$  for newborn firms.

## 2.1 Solution of the firm's problem

Samaniego (2006a) shows in a continuous time context without uncertainty that this class of problems can be analyzed using standard recursive techniques by specifying two different control variables, investment  $u \geq 0$  and disinvestment  $h \geq 0$ , and setting

$$k = k_{-1} + u - h. \quad (2)$$

Extending this insight to our environment, we are able to prove the following:

**Lemma 1** *The solution to the problem in equation 1 is the same as the solution to one where  $k$  is not a control variable and is instead determined by 2.*

**Proposition 1** *Optimal capital  $k^*(z, k_{-1}; v_t)$  is characterized by two thresholds  $\bar{k}^*(z, v) <$*

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<sup>5</sup>Note that since productivity  $z_t$ , wages and prices will enter the firm's investment decisions multiplicatively, it is without loss of generality to assume that any two of them are constant over time as long as their evolution is described by a Markov process.

$\underline{k}^*(z, v)$  that do not depend on  $k_{-1}$  such that:<sup>6</sup>

$$k^*(z, k_{-1}; v_t) = \begin{cases} \underline{k}^*(z, v) & \text{if } k_{-1} > \underline{k}^*(z, v) \\ \bar{k}^*(z, v) & \text{if } k_{-1} < \bar{k}^*(z, v) \\ k_{-1} & \text{if } k_{-1} \in [\bar{k}^*(z, v), \underline{k}^*(z, v)] \end{cases} \quad (3)$$

The proof of Proposition 2.1 hinges on the fact that the expected future value of the firm  $\frac{1-\lambda(z)}{1+i}EV(z', k(1-\delta); v')$  does not depend at all on  $k_{-1}$ . This means that, if firms choose to invest or disinvest, the derivative of this expression will optimally be set to equal either  $-1$  or  $\kappa - 1$  respectively – which does not depend on  $k_{-1}$  either. Thus, firms whose value of  $k_{-1}$  is below a certain threshold will invest up to  $\bar{k}^*(z, v)$ , and firms whose value of  $k_{-1}$  is above a certain threshold will disinvest to  $\underline{k}^*(z, v)$ . Since  $\frac{1-\lambda(z)}{1+i}EV(z', k(1-\delta); v')$  can be shown to be decreasing in  $k$ ,  $\bar{k}^*(z, v)$  is strictly less than  $\underline{k}^*(z, v)$ . Firms in between exercise their options to neither invest nor disinvest – waiting for further information on their production opportunities  $z_t$  and volatility  $v_t$  – in which case their expected future value will fall in between these two bounds  $(-1, \kappa - 1)$ . As a result, the optimal investment rules are characterized by a "range of inaction"  $[\bar{k}^*(z, v), \underline{k}^*(z, v)]$ , where neither investment nor active disinvestment occurs.

## 2.2 Equilibrium

Define the measure  $\mu_t : Z \times \mathbb{R}^+$  to be the measure over firm types  $(z_t, k_{t-1})$  at date  $t$ . Let  $I(\cdot)$  be an indicator function that equals one if its argument is true and zero otherwise. The state of the economy evolves according to

$$\mu_{t+1}(\mathbf{z}, \mathbf{k}) = \int_{k \in \mathbb{R}^+} \int_{z' \in \mathbf{z}} \int_{z \in Z} I(k^*(z, k; v_t) \in \mathbf{k}) dF_z(z'|z, v_{t+1}) d\mu_t(z, k) \quad (4)$$

for all Borel sets  $(\mathbf{z}, \mathbf{k}) \subset Z \times \mathbb{R}^+$ . If  $Z$  is discrete then (4) can be translated accordingly so that  $\mathbf{z}$  is any number or subset of numbers in  $Z$ .

**Definition 1** *An equilibrium of the model economy is an initial condition  $(\mu_0, v_0)$  and a sequence  $\{\mu_t, v_t\}_{t=1}^\infty$  such that  $v_t$  follows the Markov process  $F_z$  and  $\mu_t$  satisfies equation 4.*

**Proposition 2** *Suppose  $\sup\{z \in Z\} < \infty$ . An equilibrium of the model economy exists such that output at all firms is finite. Also there exists  $k^{\max} < \infty$  such that  $\mu_t(\mathbf{z}, k : k > k^{\max}) = 0$  for  $t \geq 1$ .*

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<sup>6</sup>See Veracierto (2008) for a similar result in a context with asymmetric labor adjustment costs. The same applies to Proposition 2.2 below, which is an industry implication of Proposition 2.1.

**Proposition 3** *Suppose  $Z$  has finite values. Then there is a finite set  $K \subset \mathbb{R}^+$  and  $T < \infty$  such that  $\mu_t(\mathbf{z}, k : k \notin K) = 0$  for  $t \geq T$ .*

Propositions 2 – 3 have the following useful implications. If the support of  $z$  is bounded, then above a certain date the capital stocks of all firms will be bounded. This is useful for computing the model economy as we just need to identify the bounds on capital and focus on initial conditions that lie within those bounds. Notice that in our environment there is no notion of a steady state equilibrium unless  $F_v$  equals the identity matrix, i.e. the uncertainty state of the economy does not change over time. This implies that our model economy cannot be calibrated by matching certain statistics from the data to those generated in a model steady state: we will have to run large numbers of simulations of the model economy during the calibration process.

## 2.3 Growth and Uncertainty

An implication of this model is the following. Suppose that the range of inaction is not very sensitive to the value of the volatility parameter. This is likely to be the empirically relevant case: while the distribution of idiosyncratic productivity varies with the value of  $v_t$ , the overall variance in  $z_t$  likely swamps any *differences* in those variances across volatility regimes. Nonetheless, industry features may make some firms more sensitive to volatility differences than others.

Under this assumption, it should be clear that the depreciation rate  $\delta$  is going to be a key determinant of the extent to which there is mismatch between  $z$  and  $k$ . If  $z$  does not change much over time, a low value of  $\delta$  means that there will be little mismatch between  $z$  and  $k$  for a while. In times of high uncertainty there may be a bit more mismatch since productivity is more volatile. In contrast, if  $\delta$  is high, mismatch may develop rapidly. When uncertainty is low this may actually be a good thing, because when there are negative productivity shocks the firm can adjust to them by depreciating. When uncertainty is high however the rapid depreciation means that there are two factors contributing to potential mismatches, productivity volatility and rapid depreciation.

It is difficult to provide an analytical result about this because of the fact that decision rules depend on  $k_{-1}$ . Industry growth in this environment depends on the *distribution* of  $k_{-1}$ , which is an endogenous variable. Thus, we turn to quantitative analysis to test whether the above intuition holds out in a calibrated version of our model economy.



### 3 Quantitative analysis

We now calibrate the model to match a typical industry according to US data. Then we examine how industry growth depends on the presence or absence of volatility shocks.

Our calibration strategy will be somewhat unusual. Typically in a model of this type where there are counterfactual experiments one calibrates the model to match a hypothetical steady state and then performs experiments. In our case, however, our model industry switches between volatility regimes. Thus we adopt a calibration strategy that does not assume a steady state at any given point in time. Instead, we assume values for certain parameters, and match the remainder by simulating the model economy for a large number of firms over a large number of periods, comparing the moments of the model economy with the statistics we wish to match.

Since we are not calibrating a steady state, our calibration requires the simulation of the model over several periods including both high and low volatility regimes. We simulate the behavior of 1000 firms, over 1400 periods, and discard the first 400 periods to ensure any assumptions on initial conditions are washed out.<sup>7</sup> The volatility process evolves according to  $F_v$  and firm values of  $z_t$  follow  $F_z(\cdot)$ . Firm investment and therefore industry growth result from the firms following the optimal investment and employment rules described above. When a firm exits it is replaced by an entrant with a value of  $z$  drawn randomly from  $\psi(\cdot)$ . Industry growth is defined as the log growth rate in the sum of output across all the firms.

Notice that this exercise has several interpretations. One is that it measures the difference in industry growth on average in an environment where uncertainty is purely idiosyncratic, i.e. there are no aggregate shocks other than the process  $v_t$  itself. Another however is that we are looking at differences in industry growth when  $z_t$  might have common elements across firms. When taking averages over large numbers of simulations, these two will yield the same results, thus our results are informative about the impact of uncertainty purely through the idiosyncratic productivity, through aggregate productivity, or any combination of the two, net of any level shocks. This is important because in the data there are several approaches to measuring uncertainty: our simulation results should speak to all of them.

Before listing the calibration parameters we need to parameterize the distributions  $F_v$  and  $F_z$  as well as the exit function  $\lambda(\cdot)$  and the entry function  $\psi(\cdot)$ .  $F_v$  is a matrix that

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<sup>7</sup>The initial conditions assume that the volatility is low and that firm productivities are random draws from the ergodic distribution of  $F_z(\cdot, 0)$ .

describes the probability of switching between volatility regimes. We set it equal to:

$$F_v = \begin{bmatrix} 0.878 & 0.0122 \\ 0.4 & 0.6 \end{bmatrix}.$$

This matches the average duration of high and low uncertainty regimes in the data to be considered later.

To complete the calibration process we require values of the set of shocks  $Z$ . We select a large number of shocks so that inertia in firms' decisions is not mainly guided by inertia built into the calibrated model by simply having few  $z$  values. We choose 60 values<sup>8</sup> of  $z$ , log-distributed evenly between 0.5 and 2. This provides a wide range of firm sizes.<sup>9</sup> Then, we assume that  $F_z(z'|z, 0)$  is a discretized version of  $\log z_{t+1} = \log z_t + \varepsilon_{t+1}$ , and we assume that the standard deviation of  $\varepsilon_{t+1}$  equals a parameter  $\sigma$ . When volatility is high we assume that  $F_z(z'|z, 1)$  is a discretized version of  $\log z_{t+1} = \rho(z_t) \log z_t + \varepsilon_{t+1}$  and that the standard deviation of  $\varepsilon_{t+1}$  is  $\sigma(1 + \omega)$ ,  $\omega > 0$ . The factor  $\rho(z_t)$  is chosen for each  $z$  to ensure that  $\int z' dF_z(z'|z, 1) = \int z' dF_z(z'|z, 0) \forall z$ : in other words, it is set so that  $F_z(z'|z, 1)$  is a mean-preserving spread of  $F_z(z'|z, 0)$ .<sup>10</sup>

As for  $\lambda(\cdot)$ , we assume that  $\lambda(z)$  declines exponentially from  $\lambda^{\max}$  down to zero as  $z$  rises. This captures the fact that larger firms tend to exit less often. The specific functional form is  $\lambda(z) = \lambda^{\max} \frac{\log[\max(z)] - \log[z]}{\log[\max(z)] - \log[\min(z)]}$ .

For  $\psi(\cdot)$ , we assume that  $\psi(z)$  declines with  $z$ , matching the well known fact that entrants are typically smaller than incumbents. The specific functional form is  $\psi(z) = \frac{(\max(z) - z) + \bar{\psi}}{\sum_{z' \in Z} \psi(z')}$ . As  $\bar{\psi} \rightarrow \infty$ , the distribution becomes uniform.

The following parameters remain to be calibrated:  $\alpha$ ,  $\theta$ ,  $i$ ,  $\delta$ ,  $e$ ,  $w$ ,  $\bar{\psi}$ ,  $\lambda^{\max}$ ,  $\kappa$ ,  $\sigma$  and  $\omega$ .

We set  $e = w = 1$  without loss of generality. These are all essentially scale parameters and do not affect the response of the model economy to uncertainty.

We set  $\alpha = 0.63$  and  $\theta = 0.25$ , as in Samaniego (2010). We set  $i$  to equal 2 percent, a standard number in the business cycle literature.

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<sup>8</sup>Relates studies typically use under 10 values of  $z$  to conserve computational resources when there are many dimensions of heterogeneity as here. We found that working with 30 provided negligible differences to results.

<sup>9</sup>In the calibrated economy the largest firm is over a million times larger than the smallest.

<sup>10</sup>Notice that for low uncertainty we approximate a random walk, and for high uncertainty we have persistence parameters that are set to ensure uncertainty is a mean preserving spread. The reason for the random walk assumption is that, given that we are using a discretized range for  $z_t$ , there will be mean reversion regardless. When we generated several thousand values of  $z_t$  we found that the computed autocorrelation was about 0.72. This is similar to Samaniego (2010), which has lumpy investment due to an irreversible technology updating decision rather than a simple investment irreversibility as we have here.

As a benchmark we set  $\delta = 0.0827$ . This is the median value across the industries in the data to be presented in more detail later.<sup>11</sup> Later we consider  $\delta \in [0.06, 0.11]$ , which is the empirically relevant range in our data.

The volatility parameter  $\omega$  is set to equal 0.5. The literature does not provide much guidance regarding this parameter so we set it so as to be significant but not so large as for firm volatility when  $v = 1$  to be too much larger than when  $v = 0$ . We find that this value also roughly replicates the differences in the response of industry growth to volatility across industries with different values of  $\delta$  observed later in the data.<sup>12</sup>

It remains to calibrate the parameters  $\sigma$ ,  $\kappa$ ,  $\bar{\psi}$  and  $\lambda^{\max}$ . We select these parameters using simulated annealing (Bertsimas and Tsitsiklis (1993)) so as to match some key moments of the data on industry dynamics:

1. The share of entrants that are "small", i.e. one third the size of the average firm or less in terms of employment – see Samaniego (2008).
2. The 5-year exit rate of new firms – see Evans (1987).
3. The average 5-year exit rate – see Evans (1987).
4. The share of firms experiencing a lump in investment – defined as in Doms and Dunne (1998) as an investment of more than 30 percent of the current capital stock. This value is 6 percent.<sup>13</sup>

The model matches these statistics reasonably well in spite of its simplicity. We also find that the model reasonably matches the share of investment that occurs in lumps, which in the data is 25 percent. In the model it is a bit lower at 19 percent, not far off.

We find in the calibrated economy that, as proven, the disinvestment threshold  $\underline{k}^*(z, v)$  is more than the investment threshold  $\bar{k}^*(z, v)$  for any  $z, v$ . In addition we find that  $\bar{k}^*(z, 0) > \bar{k}^*(z, 1)$  for most values of  $z$ : when volatility is higher, firms invest more conservatively. See Figure 1. We also find that  $\underline{k}^*(z, 0) < \underline{k}^*(z, 1)$ : when volatility is higher, the threshold for firms to disinvest is higher (i.e they are also more conservative). In other words, the range

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<sup>11</sup>The data are based on US manufacturing industries, with depreciation measured by the US Bureau of Economic Analysis.

<sup>12</sup>In our empirical results, the coefficient on value added is such that the difference in industry growth in times of high uncertainty between the industries with the highest and lowest values of  $\delta$  is about 1.4 percent. In Figure 2 below, this difference in the model is about 1.2 percent.

<sup>13</sup>The published version of Doms and Dunne (1998) reports a different number, but entrants and exiters are excluded from that sample. Six percent is the value in the panel that includes entry and exit. This is also the median industry value in Compustat.

Table 1: Calibrated parameter values

Parameter	Value	Source
$\alpha$	0.63	Samaniego (2010)
$\theta$	0.25	Samaniego (2010)
$i$	0.02	Business cycle literature
$\delta$	0.0827	UNIDO data
$\psi$	4.5743	See text
$\lambda^{\max}$	0.1548	See text
$\kappa$	0.1	See text
$\sigma$	0.0218	See text
$\omega$	0.5	See text

Table 2: Model Statistics

Statistic	Model	Data	Source
Small entrants	0.80	0.74	Samaniego (2008)
Average exit	0.22	0.22	Evans (1987)
Entrant exit	0.37	0.37	Evans (1987)
Lumpy firms	0.05	0.06	Doms and Dunne (1998)
Lumpy investment	0.19	0.25	Doms and Dunne (1998)

of inaction is larger when volatility is high. That said, the differences are quite small. This can be seen in that in Figure 1 the slopes of the edges of the surfaces have a similar slope to the volatility axis line: the range of inaction is not obviously very different – at least for the benchmark parameterization.

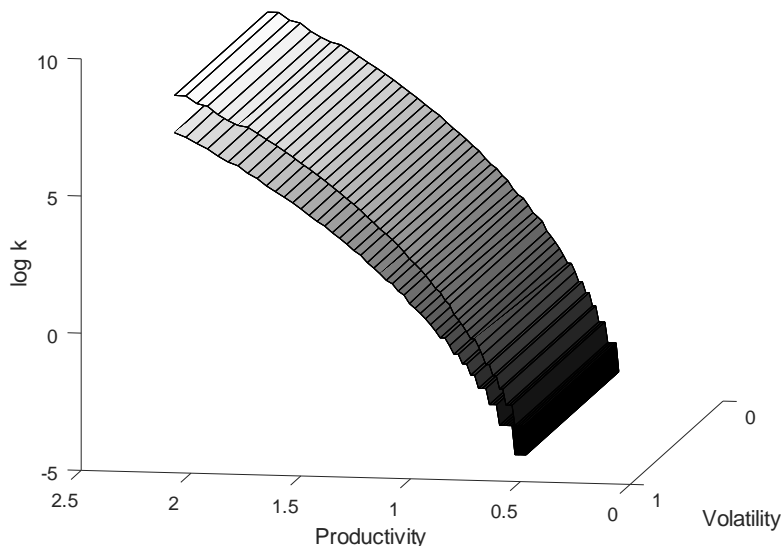


Figure 1 – Decision rules. The upper surface is the disinvestment threshold  $\bar{k}^*(z, v)$  for given values of  $z$  and  $v$ . The lower threshold is the investment threshold  $\underline{k}^*(z, v)$  for given values of  $z$  and  $v$ .

### 3.1 Experiments

Having calibrated these parameters we measure industry growth in the simulated economy in high and low volatility regimes. We compute this separately for a variety of different values of  $\delta$ . We define the empirically relevant range of  $\delta$  as that we observe in the data to be presented later, from about 6 percent to about 11 percent. We compute this by running the experiences of a large number of firms as described earlier. It is worth underlining that, while our volatility process is described as being aggregate and having an impact on idiosyncratic productivity, the same statistics obtain if all firms have common shocks.

Figure 2 displays how growth in the high and low uncertainty regimes varies on average depending on  $\delta$ . Several observations stand out. First, when  $\delta$  is low firms tend to grow faster when uncertainty is high. However the differences are small. It is clear that, as hypothesized, when  $\delta$  is high industry growth is much slower when uncertainty is high.

What does Figure 2 tell us about the aggregate impact of uncertainty shocks? The median industry in terms of  $\delta$  is one where industry growth is not particularly sensitive to uncertainty shocks. Instead, the aggregate impact of uncertainty shocks will be driven by what happens particularly in the high- $\delta$  industries. The model suggests that these industry differences are an important part of the impact of uncertainty on the aggregate economy.

Another implication is that the impact of uncertainty on different economies will depend on their *industry composition*. Countries which, for whatever reason, specialize in low- $\delta$  industries may be insensitive to uncertainty shocks, whereas countries which specialize in high- $\delta$  industries may be more sensitive to uncertainty.

As a result, Figure 2 suggests an empirical test of the model: testing whether industries with high depreciation rates display growth disproportionately slowly compared to other industries in the presence of uncertainty shocks. A test of this kind can be used to test the real options channel of the impact of uncertainty shocks on economic growth.

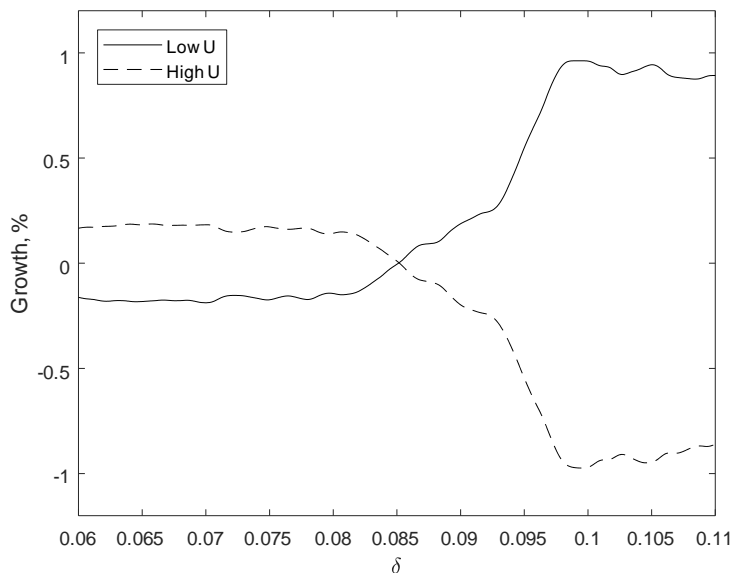


Figure 2 – Response surfaces for average growth when uncertainty is high and when uncertainty is low, as a function of  $\delta$ . The values were computed in simulations of the model economy for 1000 values of  $\delta$ .

Figure 3 suggests why this may be happening: in industries with high  $\delta$ , labor productivity growth is slower when  $v$  is high. Since labor is allocated efficiently (conditional on

$k$  and  $z$ ) this must be because capital is significantly misallocated in those industries when uncertainty is high, less so when uncertainty is low. In contrast, when  $\delta$  is high, capital depreciation within the zone of inaction is not an important factor of misallocation regardless of uncertainty.

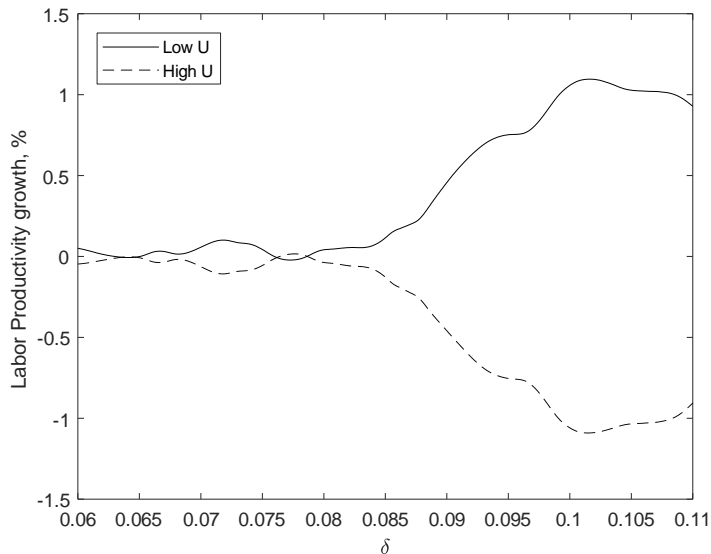


Figure 3 – Average labor productivity growth when uncertainty is high and when uncertainty is low as a function of  $\delta$ . The values were computed in simulations of the model economy for 1000 values of  $\delta$ .

Another prediction of our model is that high  $\delta$  is related to more frequent lumps in investment, because it leads to more frequent mismatches between productivity and capital. Figure 4 illustrates this prediction in the context of the model. This suggests that another way to test our mechanism would be to see whether in the data industries with higher  $\delta$  also experience more frequent investment lumps, as in Figure 4.

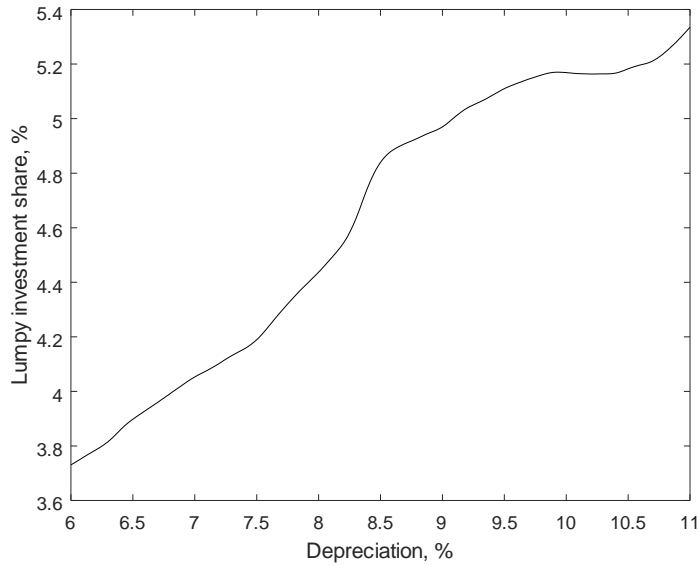


Figure 4 – Relationship between depreciation and the share of firms experiencing an investment lump on average in the calibrated economy.

Finally, if high  $\delta$  is related to high lumpiness, we have an additional prediction we might use to test the real options channel of the impact of uncertainty shocks on economic growth: seeing whether industries with high *investment lumpiness* grow disproportionately slowly compared to other industries in the presence of uncertainty shocks. Indeed, Figure 5 shows that the calibrated economy displays this feature.

In the remainder of the paper, we will test the predictions found in Figures 2 – 5:

1. lumpiness and depreciation should be related across industries;
2. industries with high depreciation should grow disproportionately slowly in the presence of uncertainty shocks; and
3. industries with high lumpiness should grow disproportionately slowly in the presence of uncertainty shocks.



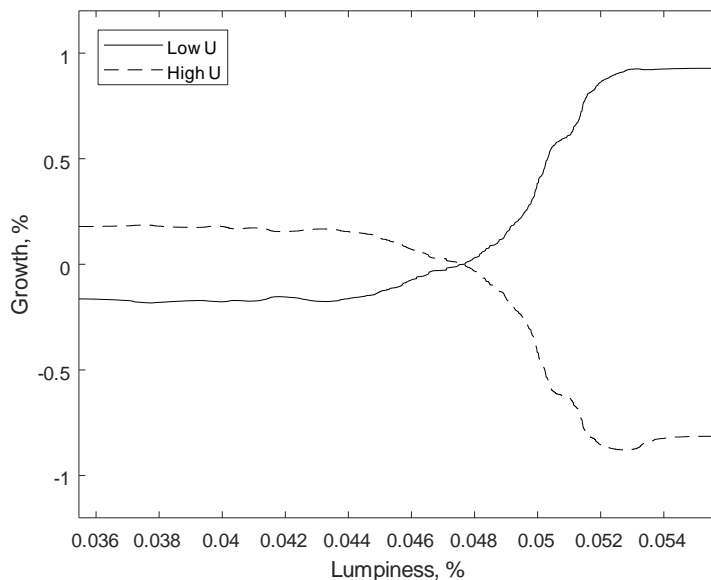


Figure 5 – Response surfaces for average growth when uncertainty is high and when uncertainty is low, as a function of lumpiness. The values were computed in simulations of the model economy for 1000 values of  $\delta$ . The graph represents different combinations of observed lumpiness and industry growth.

Finally, it is worth noting that these effects are for empirically reasonable values of depreciation rates. Suppose instead that  $\delta = 1$ . In this case, since capital depreciates fully every period, no firm ever pays the irreversibility cost  $\kappa$ . Thus, there is no scope for misallocation (given that parameter value), and firms set capital  $k$  to equal their optimal values (conditional on  $\delta$ ) – i.e. there is no range of inaction. Substituting this into the production function, the production function will be a function of  $z^{\frac{1}{1-\alpha}}$ , i.e. a convex function of  $z$ . It is easy to show that the expected value of output under such circumstances is increasing in uncertainty. Thus firms will tend to grow faster in a high uncertainty regime relative to a low uncertainty regime.

More formally, suppose that  $\delta = 1$ , and that the productivity process for  $z_{t+1}$  does not depend on  $z_t$ . In this case  $k$  will be chosen optimally so that  $k^*(z, v) \propto z^{\frac{1}{1-\alpha}}$ , and output

will be  $xz^{\frac{1}{1-\alpha}}$ , where  $x$  is a constant. As a result, average output will be

$$Y(v) = \int xz^{\frac{1}{1-\alpha}} dF_z(z, v) = x \int z^{\frac{1}{1-\alpha}} dF_z(z, v)$$

We have that  $\int z dF_z(z, 0) = \int z dF_z(z, 1)$  by assumption. Observe that  $z^{\frac{1}{1-\alpha}}$  is a convex transformation of  $z$ : thus,  $Y(0) < Y(1)$  and the economy will grow when volatility increases (and shrink when volatility decreases): uncertainty promotes growth. If we allow the productivity process for  $z_{t+1}$  to depend on  $z_t$ , the economy would transit more slowly between the ergodic distributions of  $F_z(\cdot|z, 0)$  and  $F_z(\cdot|z, 1)$  (which will be a mean-preserving spread of  $F_z(\cdot|z, 0)$ ) but the environment with  $v = 1$  would experience faster growth than when  $v = 0$  along this transition.

Indeed, Figure 6 shows that this is the case: as  $\delta$  approaches unity, uncertainty becomes *beneficial* to growth. This is consistent with Oi (1996), Hartman (1972) and Abel (1983), who find that when firms can adjust in the face of uncertainty, uncertainty may benefit investment and growth. What happens is that for empirically relevant ranges of  $\delta$  the inflexibility induced by the adjustment cost  $\kappa$  is significant, so the misallocation effect dominates. It is interesting that the value of  $\delta$  above which Oi-Hartman-Abel effects dominate is about 0.4, well outside the empirically relevant range. In this paper the empirically relevant range is about 0.06 to 0.116, based on our manufacturing data. The turning point in Figure 6 after which the negative interaction of uncertainty and growth starts to weaken is around 0.14, again outside the empirically relevant range.

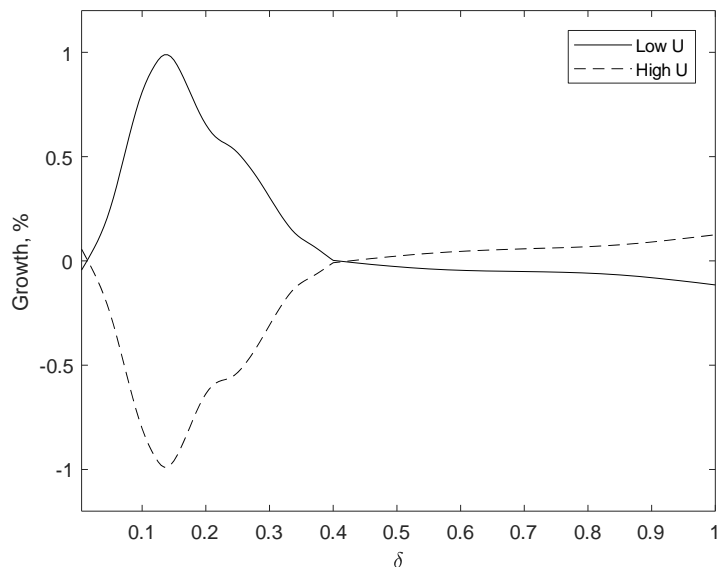


Figure 6 – Response surfaces for average growth when uncertainty is high and when uncertainty is low, as a function of  $\delta$ . The values were computed in simulations of the model economy for 1000 values of  $\delta$ .

## 4 Empirical strategy

Our objective is to see whether certain industry technological characteristics – namely capital depreciation rates and lumpiness rates – lead industries to be more sensitive to uncertainty shocks. To do so, we estimate the following equation:

$$Growth_{c,i,t} = \delta_{i,c} + \delta_{i,t} + \delta_{c,t} + \beta(UncertaintyShock_{c,t-1} \times X_i) + \alpha Controls_{i,c,t} + \epsilon_{c,i,t} \quad (5)$$

In equation (5),  $Growth_{c,i,t}$  is a measure of growth in industry  $i$  in country  $c$  at date  $t$ . The dummy variables  $\delta_{i,c} + \delta_{i,t} + \delta_{c,t}$  capture all date- or country-specific factors that might affect growth in industry  $i$ , or factors affecting overall growth in country  $c$  at a particular date, including all economy-wide shocks. All that remains are factors that specifically affect growth in industry  $i$  in country  $c$  at date  $t$ .

$X_i$  is a technological factor of interest that characterizes the production function of industry  $i$  (such as the depreciation rate), and which is hypothesized to interact with uncertainty.

It appears in equation (5) interacted with  $UncertaintyShock_{c,t-1}$ , which is an uncertainty shock measured at date  $t - 1$ . Thus the coefficient  $\beta$  is the differential impact of industry characteristic  $X_i$  on industry growth when uncertainty in the previous year is high. Pooling data for many industries, years and countries gives our estimation strategy more statistical power and gives our results more generality.

Since  $\beta$  captures the difference in industry growth in uncertain times relative to normal times for industries with different levels of  $X_i$ ,  $\beta \neq 0$  indicates that growth in industries with high  $X_i$  is more sensitive to uncertainty. For example, if  $X_i$  measures the depreciation rate of capital, then  $\beta < 0$  would indicate that industries that use rapidly depreciating capital grow particularly *slowly* when there is uncertainty. Conversely  $\beta > 0$  would indicate that such industries grow particularly *fast* when there is uncertainty.

Our control variables  $Controls_{i,c,t}$  include an interaction ( $LevelShock_{c,t-1} \times X_i$ ). The variable  $LevelShock_{c,t-1}$  is a country- and year-specific measure of the level of economic activity at date  $t - 1$ . We interact it with the technological variable  $X_i$  also because, as is well known in the literature, increases in uncertainty may coincide with downturns in economic activity, and the level effects may interact with technological variables too. Thus we wish to condition on first moment measures of the level of economic activity. The overall level is already captured by the dummy  $\delta_{c,t}$ , so the coefficient  $\beta$  captures any residual industry-specific impact of level shocks (including the impact of uncertainty shocks on levels of overall economic activity) on industry growth based on technological measure  $X_i$ .

The need to condition on level shocks raises the possibility of endogeneity: the level and uncertainty effects may be correlated and also endogenous. See Baker and Bloom (2013). One way we handle this is precisely by looking at industry growth rather than aggregate growth. Any omitted variables that cause both growth and uncertainty (as well as level shocks) should be picked up by the  $\delta_{c,t}$  indicators. In addition, we condition on possible interactions of level effects and the technological variables. The potential endogeneity of uncertainty is already controlled for because specification (5) is based on *past* uncertainty, and current year growth cannot cause past uncertainty. We also deal with the possibility of any residual endogeneity in *industry* growth by using instrumental variables, as suggested in Baker and Bloom (2013) in the context of aggregate growth. In this case, both level and uncertainty shocks would need to be instrumented. Given a set of instruments for the level and moment shocks  $Instr(c, t - 1)$ , the instruments to be used when the instrumented variable is an *interaction* with the level and moment shocks as in specification (5) are of the form  $Instr(c, t - 1) \times X(i)$ , see Wooldridge (2002).

Since the number of group-specific effects in this estimation equation is very large,<sup>14</sup> the computational cost of estimating (5) is significant. Instead, we proceed by subtracting from all dependent and independent variables the mean value for each  $(c, t)$ ,  $(i, t)$  and  $(c, i)$  pair so that the dummy variables  $\delta_{i,c}$ ,  $\delta_{i,t}$  and  $\delta_{c,t}$  are removed from the estimation equation. We call these variables  $\widehat{Growth}_{c,i,t}$ ,  $(\widehat{UncertaintyShock}_{c,t-1} \times X_i)$  and  $\widehat{Controls}_{c,i,t}$ . Then, we estimate (5), using the de-meanded variables, and without  $\delta_{i,c} + \delta_{i,t} + \delta_{c,t}$  among the regressors. In the Appendix we show that this is equivalent to estimating the following specification:

$$\widehat{Growth}_{c,i,t} = \beta(\widehat{UncertaintyShock}_{c,t-1} \times X_i) + \alpha\widehat{Controls}_{c,i,t} + \epsilon_{c,i,t} \quad (6)$$

To estimate (5) using instrumental variables, we use the well known two stage least squares approach to instrumental variables estimation (TSLS). This involves regressing the endogenous dependent variables on the others, including dummies and instruments. We must thus modify the demeaned specification (6) so as to implement the TSLS procedure. Since the TSLS procedure requires that the large number of dummy variables should be included at both stages, we apply the demeaning procedure at both stages in order to deal with them – see Appendix for derivations. We also use limited information maximum likelihood (LIML), finding similar results (see Appendix).

The exact error structure for this procedure is not known so we use a variety of approaches, finding that the results are robust. These methods include bootstrapping, allowing for heteroskedasticity using the Huber-White method, clustering by industry, and allowing for autocorrelated errors.<sup>15</sup> The results reported use bootstrapped errors.

Country- or date-specific factors that affect a given industry will be absorbed by the indicator variables in equation (5) – including the impact of uncertainty on overall growth. Then, any interaction between uncertainty and  $X_i$  indicates that characteristic  $X_i$  is important for understanding how uncertainty shocks impact industry growth.

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<sup>14</sup>Since there are about 60 countries, 28 industries and 42 years, we would have over 50,000 fixed effects in a balanced panel.

<sup>15</sup>Bertrand et al (2004) argue that differences-in-differences specifications may suffer from problems with autocorrelated errors. However this relates to specifications where there is a persistent treatment vs. non-treatment variable. In our context there is no such problem because of the constellation of country-time and industry-time dummies. When we estimate the specification allowing for autocorrelated errors the estimated autocorrelation coefficient is small, around 0.01.

## 4.1 Discussion

Some further comments on our estimation strategy are in order. First, we seek industry technological indicators  $X_i$  that are representative of the technology of production across countries. Suppose for example that  $X_i$  represents the frequency of lumpy investment. The identification strategy does not require measures of the *observed* lumpiness at firms in industry  $i$  in each country, nor at each date. Observed lumpiness is not a strictly technological variable, as it may be affected by current economic conditions such as the level of uncertainty at date  $t$  in country  $c$ , or by country conditions including the frequency of uncertainty shocks in country  $c$ . Instead, we seek a benchmark measure of lumpiness that firms in industry  $i$  would adopt in a relatively undistorted environment – which, when distorted by uncertainty in country  $c$  at date  $t$ , might particularly impact firms in industry  $i$ . Following the related literature, we will measure the technological variables  $X_i$  such as depreciation using US data and, where possible, using data on publicly traded firms in the US, whose technological choices are unlikely to be distorted by financing difficulties or by other frictions in normal times – see Rajan and Zingales (1998), Ilyina and Samaniego (2011) and Samaniego and Sun (2015) among others.<sup>16</sup>

We also explore whether our industry-based strategy finds evidence of any important role for financial markets in either the origination or propagation of uncertainty shocks, a key question in the literature.<sup>17</sup> This is important because of our focus on depreciation and on investment lumpiness. Theory suggests that industries with rapid depreciation and where capital is more likely to be firm-specific are also those where the ability to use capital as collateral to raise external funds is weakest – see Hart and Moore (1994). In addition, Ilyina and Samaniego (2011) find that investment lumpiness is linked to external finance dependence. As a result, the risk-aversion theory of uncertainty might also predict that these industry variables interact with uncertainty shocks.

We verify that our results are not due to the risk-aversion theory in four ways. First, we include a measure of *external finance dependence* (Rajan and Zingales (1998)) in our list of technological variables. Second, we also look at R&D intensity, which Ilyina and Samaniego

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<sup>16</sup>Even so, we do find that our technical measures are correlated across time and space. Regarding time, Ilyina and Samaniego (2011) show that the rankings of industries according to most of our industry measures computed by decades persist over the period (1970-2000). Regarding country variation, the data simply do not exist to measure industry characteristics in each country separately – except for labor intensity or " $LAB_i$ ". We computed  $LAB_{c,i}$  for each country  $c$  and industry  $i$  following the procedure described later. Then for each country we computed the cross-industry correlation between  $LAB_{c,i}$  and  $LAB_i$  as measured in the US – our technological measure. We found that this correlation was positive and statistically significant at the 5 percent level in 49 out of the 54 countries for which data were available.

<sup>17</sup>See for example Arellano et al (2012) or Gilchrist et al (2014).

(2011) argue is the technological basis of external finance dependence. Third, we use labor intensity and asset fixity as additional technological variables that the literature finds to be sensitive to financial frictions. Fourth, we condition on an interaction of technology with financial crisis indicators (Laeven and Valencia (2013)). These might be expected to interact with depreciation and lumpiness too if financial channels are important.

## 5 Data

### 5.1 Defining Uncertainty

We require a measure of uncertainty that can be measured for many countries. We follow Baker and Bloom (2013) in defining uncertainty using data from bond markets. We do this because we are using data from a broad selection of countries, so that bond yields (largely based on government bond data) provide information about overall country conditions.<sup>18</sup> Uncertainty is measured using the average quarterly volatility of daily percentage changes in bond yields. Bond market volatility we view as capturing uncertainty concerning safe assets, possibly indicating the undiversifiable or unhedgable portion of uncertainty, including economy-wide uncertainty e.g. uncertainty stemming from the sovereign’s policy or default decisions. Thus we also refer to it as *systemic* uncertainty.<sup>19</sup>

### 5.2 Instrumental variables

As mentioned, there is some concern in the literature that level shocks and second moment shocks (uncertainty) could be jointly determined. This is one factor motivating our differences-in-differences specification with a complete constellation of  $(i, c)$ ,  $(i, t)$  and  $(c, t)$  dummy variables: endogeneity between aggregate first and second moment shocks is controlled for, only effects that are specific to industries in a particular country in periods of

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<sup>18</sup>Baker and Bloom (2013) also report other measures of uncertainty, based on stock market volatility, cross-sectional return dispersion and exchange rate volatility. We used those measures and found no robust interactions with industry variables. This is not surprising since stock markets are not very deep in many developing economies, and since vulnerability to external shocks likely varies across countries. Thus bond market uncertainty is likely to be the best indicator of volatility for a broad set of countries.

<sup>19</sup>Jurado et al (2015) measure uncertainty using the component of macroeconomically important measures that is unforecastable based on a wide array of time series. We do not adopt this approach to measuring uncertainty as such an approach requires a large set of time series to identify unforecastable events, which would be challenging to perform in a consistent manner for many countries. In any case, we find that, for the US, the annual macroeconomic uncertainty series of Jurado et al (2015) at a quarterly frequency has a correlation with the Baker and Bloom (2013) bond market uncertainty series of 0.27, statistically significant at the 1 percent level.

uncertainty such as the interaction of second moment shocks and technology will be picked up by our interaction coefficients. Rajan and Zingales (1998) introduce the methodology for this reason, albeit in a context and without a time panel.

We also account for endogeneity by using a standard instrumental variables procedure. We employ instruments that have been found to be appropriate in the related literature. Specifically, Baker and Bloom (2013) use measures of exogenous "disasters" as instruments – see their paper for further details:

1. Natural Disasters: Extreme weather and geological events as defined by the Center for Research on the Epidemiology of Disasters (CRED). Industrial and transportation disasters are not included.
2. Terrorist Attacks: high casualty terrorist bombings as defined by the Center for Systemic Peace (CSP).
3. Political Shocks: An indicator for successful assassination attempts, coups, revolutions, and wars, from the Center for Systemic Peace (CSP) Integrated Network for Societal Conflict Research. There are two types of political shocks: forceful or military action which leads to the change of executive authority within the government, and a revolutionary war or violent uprising led by politically organized groups outside current government within that country.

Each of these country-year indicator variables is interacted with the industry technological measure of interest. This interaction variable is the relevant instrument in our context, where the independent variables to be instrumented (uncertainty shocks times industry characteristics) are themselves interaction variables, see Wooldridge (2002).<sup>20</sup>

### 5.3 Industry outcomes

We measure  $Growth_{c,i,t}$  in three ways: (1) the log change in industry value added, as reported in the INDSTAT3 and INDSTAT4 databases, distributed by UNIDO; (2) the log change in

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<sup>20</sup>As mentioned, Baker and Bloom (2013) test the validity of these instruments with respect to aggregate growth. Sargan tests confirmed the statistical validity of our interaction instruments with respect to uncertainty and industry growth. The exception is with one of our 3 industry outcome measures, index growth. In this case, the instruments were valid (and results were similar) if we dropped one of our four interaction instruments, terrorism shocks.

An econometric procedure that is robust to weak instruments is LIML. We repeated our estimation using LIML, finding almost identical results. See Appendix.



gross output; and (3) the log change in the Laspeyres production index. Having three different growth measures gives the results considerable robustness.

These variables are reported for 28 manufacturing industries based on the ISIC-revision 2 classification in INDSTAT3. We use only countries for which there are at least 10 years of observations. To avoid the influence of outliers, the 1st and 99th percentiles of  $Growth_{c,i,t}$  are eliminated from the sample. We lose some countries as uncertainty data in Baker and Bloom (2013) are not available for the whole globe. This generates a sample of 60 countries from 1970 to 2012. The panel is unbalanced, and the sample sizes vary across countries and industries as some of the data were not reported by national statistical agencies. The Appendix lists the country sample and the number of observations for each country. Data from 1970 to 2004 are from INDSTAT3, while data from 2005 to 2012 are from the successor dataset INDSTAT4. The United States is not included in the regressions because it is the benchmark economy for measuring industry technological variables.

## 5.4 Industry Technological Measures

Our exercise requires a definition of “technology.” Since the work of Kydland and Prescott (1982), theoretical business cycle analysis is commonly performed within the context of models of economic growth. We follow the conventions of growth theory by defining “technology” in terms of the production function. We identify industry differences in the production technology using factor intensities, or using the qualitative attributes of factors of production, an approach that dates back to at least the seminal work of Cobb and Douglas (1928). For example, differences between the technology for producing Food Products (ISIC 311) and the technology for producing Transport Equipment (ISIC 381) can be described in terms of the former being less R&D intensive and less labor-intensive than the latter. Our technology indicators include measures of labor intensity, R&D intensity, asset fixity, capital depreciation, and the lumpiness of investment.

The different technological measures are calculated using U.S. data and are assumed to represent real industry technological characteristics in a (relatively) unregulated and financially frictionless environment. Technological differences among industries are assumed to be persistent across countries, meaning that the rankings of these indices are stable across countries (or would be in the absence of uncertainty shocks), although index values in each country do not necessarily have to be the same.<sup>21</sup> See Rajan and Zingales (1998), Ilyina and

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<sup>21</sup>The measures below are drawn from Ilyina and Samaniego (2011) and Samaniego and Sun (2015), and represent averages over the period 1970-2000. Industry measures computed using the Compustat database

Samaniego (2011) and Samaniego and Sun (2015) for related discussions.

- *Capital depreciation*: we compute the industry rate of depreciation ( $DEP_i$ ) using the BEA industry-level capital flow tables.
- *Investment lumpiness*: As in Ilyina and Samaniego (2011), lumpiness ( $LMP_i$ ) is defined as the average number of investment spikes per firm during a decade in a given industry, computed using Compustat data. A spike is defined as an annual capital expenditure exceeding 30% of the firm's stock of fixed assets, as in Doms and Dunne (1998).

In addition to these variables which are suggested by the model, to account for the possibility that we are capturing financial channels rather than real options channels we include some variables that the related literature has found to be related to the intensity of financing constraints:

- *External finance dependence*: Many studies such as Rajan and Zingales (1998) find that the industry tendency to draw on external funds is related to growth and/or the business cycle. As such, any interaction of this variable with uncertainty could indicate the importance of financial channels for the propagation of uncertainty shocks. We measure external finance dependence ( $EFD_i$ ) as the share of capital expenditures not financed internally, see Rajan and Zingales (1998) for details.
- *R&D intensity*: R&D intensity is closely related to finance dependence (Ilyina and Samaniego (2011, 2012)), so it could interact with uncertainty if financial sources or channels are important. R&D intensity ( $RND_i$ ) is measured as R&D expenditures over total capital expenditures, as reported in Compustat.
- *Asset fixity*: Braun and Larraín (2005) argue that asset fixity is a key determinant not of the need for external finance but of the *ability* to raise external funds, so an interaction of fixity with uncertainty could be indicative of financial sources or channels for uncertainty. Asset fixity ( $FIX_i$ ) is the ratio of fixed assets to total assets, computed using Compustat data following Braun and Larraín (2005).
- *Labor intensity*: Hart and Moore (1994) argue that human capital is inalienable and thus is related to the ability to borrow. Samaniego and Sun (2015) find that labor intensive industries are more sensitive to the business cycle, particularly in financially

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are median firm values for each industry unless otherwise stated.

Table 3: Correlation Matrix of Industry Technological Characteristics

Industry variable	EFD	DEP	RND	LAB	FIX
DEP	.09	–	–	–	–
RND	.79**	.09	–	–	–
LAB	.05	.39**	.17	–	–
FIX	.09	.19	.39**	.22	–
LMP	.50**	.39**	.61**	.31	.73**

Note:  $EFD_i$  (external finance dependence),  $DEP_i$  (depreciation),  $RND_i$ (R&D intensity),  $LAB_i$  (labor intensity),  $FIX_i$  (fixity) and,  $LMP_i$  (investment lumpiness) are the average of 70s, 80s and 90s from Ilyina and Samaniego (2011). \*\* significance level 5%

underdeveloped economies, so that if financial channels are important for uncertainty we might expect labor intensity to interact with uncertainty as well. As in Ilyina and Samaniego (2011), labor intensity ( $LAB_i$ ) is measured using the ratio of total wages and salaries over the total value added in the US, using UNIDO data.

Table 3 shows the matrix of correlations among the technological measures. Notably,  $DEP_i$  and  $LMP_i$  are positively correlated, as predicted by the model. Depreciation and lumpiness are correlated with some other variables, however, so it is not ex ante clear how to interpret an interaction of a particular technological variable with uncertainty – unless the interactions of its correlates are not significant or not robust.

## 5.5 Control variables

In the empirical literature on industry growth it is common to condition on the share of industry  $i$  in manufacturing in the previous period, to control for mean reversion, structural change, or other secular factors of industry growth. We do so too.

Given the likely correlation between first and second moment shocks, we condition on interactions of the technological variables with first moment shocks as well. Since uncertainty is measured using bond yield volatility, the first moment shock is the average daily 10-year Government bond yield. In addition, Samaniego and Sun (2015) find that technological characteristics may interact with *contractions*, so we condition on interactions of contractions and the technological variables as well, as a non-linear control for business cycle effects. Contractions are defined using a standard peak-trough criterion as implemented by the NBER, see Samaniego and Sun (2015) for details. Our results concerning uncertainty turn out not to be sensitive to the presence of this control.

As mentioned, in one of our robustness exercises we condition on whether or not the interactions of interest are robust to including an interaction of technology with a *financial crisis indicator*. We draw on the Systemic Banking Crises Database developed by Laeven and Valencia (2013), which covers the period 1970 to 2011. We define the variable  $Crisis_{c,t}$  to equal one if the Database considers country  $c$  at date  $t$  to be experiencing a banking crisis, and zero otherwise. A year-country pair is determined to be in crisis if there are significant signs of financial distress in the banking system (bank runs, significant bank losses or bank liquidations, and if there is significant policy intervention in response to losses in the banking system). Then, we use  $Crisis_{c,t} \times X_i$  as a control for each technological variable  $X_i$ , to see whether the results are driven by crises rather than uncertainty and to see whether there are financial channels for uncertainty.

## 6 Findings

### 6.1 Empirical results

We estimate the basic regression equation (5) using the three measures of industry growth as the dependent variable and inserting the interaction terms of uncertainty with the technological variables one by one. Results are in Table 4. There are several statistically significant interactions of technology indicators with systemic uncertainty. However, the only technological variables that interact *robustly* with uncertainty – in the sense that there is a significant interaction regardless of the measure of industry growth – are depreciation  $DEP_i$  and investment lumpiness  $LMP_i$ . We conclude that the key interactions of interest are between uncertainty and these two technological variables,  $DEP_i$  and  $LMP_i$ , as found in the model. We also find that other technological interactions associated with financial frictions are not robust in that they either have inconsistent sign or inconsistent statistical significance: for example we do not find that external finance dependence ( $EFD_i$ ) interacts with uncertainty shocks. In addition, Ilyina and Samaniego (2011) argue that the deep technological characteristic underlying external finance dependence is in fact R&D intensity: we do not find evidence of an interaction between uncertainty and  $RND_i$  either. Nor do we find interactions with the financial ability variables,  $LAB_i$  and  $FIX_i$ . We thus conclude that real options considerations – rather than financial frictions – are responsible for these results. The absence of evidence linking uncertainty with financial dependence is consistent with the findings of Caldara et al (2016), who find that while uncertainty may sometimes have an impact on financial markets they are not the main source thereof.

Table 4: Basic Results

This table represents results from the following regression:

$$Growth_{c,i,t} = \delta_{i,c} + \delta_{i,t} + \delta_{c,t} + \beta(UncertaintyShock_{c,t-1} \times X_i) + \alpha Controls_{i,c,t} + \epsilon_{c,i,t}$$

We only report  $\beta$ . Each cell represents one regression. The dependent variable is industry value added growth rate, output index growth rate and gross output growth rate. Independent variables are the following:  $EFD_i$ (external finance dependence),  $DEP_i$ (depreciation),  $RND_i$ (R&D intensity),  $LAB_i$ (labor intensity),  $FIX_i$ (fixity) and  $LMP_i$ (investment lumpiness), are the average of 70s, 80s and 90s from Ilyina and Samaniego (2011). Standard errors in parentheses, \*\*\* p<0.01, \*\* p<0.05.

	Industry growth measure $Growth_{c,i,t}$		
$X_i$	Value added	Output index	Output
DEP	-.284*** (.0576)	-.00679*** (.00258)	-.139** (.0540)
LMP	-.676*** (.189)	-.0233*** (.0060)	-.297*** (.0765)
EFD	-.256* (.144)	-.0134* (.0060)	.0201 (.0420)
RND	-.188 (.127)	-.0243** (.0099)	-.145** (.0724)
LAB	-3.709*** (1.275)	-.0167 (.0329)	-.788 (.554)
FIX	2.163* (1.162)	.108*** (.0381)	1.263** (.534)
Obs	16,149	15,115	16,152

Another way to see whether our findings regarding  $LMP_i$  and of  $DEP_i$  are related to financial constraints – as opposed to real options considerations – is to compare our uncertainty measures with the financial crisis indicator  $Crisis_{c,t}$ . We find that the correlation between  $Crisis_{c,t}$  and uncertainty is quite high. The correlation is 9.29 percent and very highly statistically significant. The relationship remains highly statistically significant even when we condition on country fixed effects. This suggests that there could be a finance-uncertainty link. Then, we introduce into our specification an additional control in the form of an interaction variable of the technological variables with the financial crisis indicator  $Crisis_{c,t}$ . The specification becomes:

$$\begin{aligned} Growth_{c,i,t} = & \delta_{i,c} + \delta_{i,t} + \delta_{c,t} + \beta(UncertaintyShock_{c,t-1} \times X_i) \\ & + \beta_C(Crisis_{c,t} \times X_i) + \alpha Controls_{i,c,t} + \epsilon_{c,i,t} \end{aligned}$$

If financial frictions, not real options, are the true cause of our observed interactions, we might expect these measures to interact with crises, or expect that adding the crisis interaction as a control variable might reduce the statistical significance of our coefficient of interest  $\beta$ .

We find that, first of all, the impact on industry growth of the interaction of  $LMP_i$  and of  $DEP_i$  with systemic uncertainty is robust to the inclusion of this control variable, indeed it remains statistically significant for all three measures of industry growth. See Table 5. In contrast, the interactions of  $Crisis_{c,t}$  with  $LMP_i$  and with  $DEP_i$  are not significant. Thus, our findings are not due to uncertainty coinciding with financial crises, nor are these interactions likely due to financial frictions.

Finally, recall that in the model these results obtain because the misallocation emerging from the interaction of uncertainty and depreciation slows labor productivity. We define labor productivity in the UNIDO data as value added in each industry divided by the number of employees. Table 6 shows that indeed for depreciation and lumpiness there is a negative interaction: those industries have disproportionately low labor productivity growth in uncertain times, providing further evidence of the misallocation effect that results from the interaction of depreciation and uncertainty.

## 7 Conclusion

We develop a canonical model of investment irreversibility and argue that it is a natural consequence of such models for growth to be more sensitive to uncertainty shocks in industries where depreciation is rapid. In addition, in such industries we would expect investment to

Table 5: Controlling for Banking Crises

This table represents results from the following regression:

$$Growth_{c,i,t} = \delta_{i,c} + \delta_{i,t} + \delta_{c,t} + \beta(UncertaintyShock_{c,t-1} \times X_i) + \beta_C(Crisis_{c,t} \times X_i) + \alpha Controls_{i,c,t} + \epsilon_{c,i,t}$$

We only report  $\beta$  and  $\beta_C$ . Each cell represents one regression. The dependent variable is value added, output index and output growth rate respectively. Independent variables are  $DEP_i$  (depreciation) and  $LMP_i$  (investment lumpiness), are the average of 70s, 80s and 90s from Ilyina and Samaniego (2011). Standard errors in parentheses, \*\*\* p<0.01, \*\* p<0.05.

$Growth_{c,i,t}$	Interaction	Industry variable $X_i$	
		DEP	LMP
Value added	$Crisis_{c,t} \times X_i$	-.0829 (.0981)	.0256 (.273)
	$UncertaintyShock_{c,t} \times X_i$	-.295*** (.0550)	-.702*** (.208)
Output index	$Crisis_{c,t} \times X_i$	.00234 (.00381)	.00147 (.0105)
	$UncertaintyShock_{c,t} \times X_i$	-.00668*** (.00254)	-.0234*** (.00727)
Output	$Crisis_{c,t} \times X_i$	.0220 (.0717)	.285 (.242)
	$UncertaintyShock_{c,t} \times X_i$	-.142** (.0610)	-.303*** (.0904)

Table 6: Labor Productivity Results

This table represents results from the following regression:

$$LabProdGrowth_{c,i,t} = \delta_{i,c} + \delta_{i,t} + \delta_{c,t} + \beta(UncertaintyShock_{c,t-1} \times X_i) + \alpha Controls_{i,c,t} + \epsilon_{c,i,t}$$

We only report  $\beta$  and  $\beta_C$ . Each cell represents one regression. The dependent variable is labor productivity growth. Independent variables are the following:  $DEP_i$  (depreciation) and  $LMP_i$  (investment lumpiness), are the average of 70s, 80s and 90s from Ilyina and Samaniego (2011). Standard errors in parentheses, \*\*\* p<0.01, \*\* p<0.05.

$X_i$	Coefficient
DEP	-17.58*** (3.428)
LMP	-44.91*** (8.915)
Obs	16,006

be more lumpy, a key mechanism through which this effect occurs that industries with rapid depreciation experience disproportionately low labor productivity when uncertainty is high. We then use a differences-in-differences specification to show that industry growth data from a large set of countries are consistent with these predictions. We conclude that the misallocation introduced by investment irreversibilities are an important mechanism through which uncertainty has an impact on economic outcomes.

More broadly, our study provides an anatomy of how uncertainty affects different parts of the macroeconomy, in order to better understand the aggregate impact of economic uncertainty. The interaction of irreversibilities with depreciation and lumpiness is also something that could be useful in future studies trying to identify real options effects, and the general strategy of looking at industry interactions to identify microeconomic features that are difficult to measure directly (such as irreversibilities) could be useful more broadly to explore topics other than the interaction of uncertainty and irreversibilities.

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# Technical Appendix (not for publication)

## A Econometric procedure

We estimate a case of the following model

$$Y_{ict} = \delta_{i,c} + \delta_{i,t} + \delta_{c,t} + X_{ict}\beta + \varepsilon_{ict} \quad (7)$$

where  $i$  is industry,  $c$  is a country,  $t$  is a year. The coefficients  $\delta_{i,c}$ ,  $\delta_{i,t}$  and  $\delta_{c,t}$  are regression coefficients on indicator variables for  $(i, c)$ ,  $(i, t)$  and  $(c, t)$  pairs respectively. We have that  $c \in \{1, C\}$ ,  $t \in \{1, T\}$  and  $i \in \{1, N\}$ . Also, the panel is unbalanced, so the number of observations is not  $C \times T \times N$ .  $C$  is the total number of countries,  $T$  year and  $N$  the total number of industries.  $X_{ict}$  is a vector of independent variables  $[X_{ict1} X_{ict2}\dots]'$ .

In order to estimate (7), we transform it so as to eliminate  $\delta_{i,c}$ ,  $\delta_{i,t}$  and  $\delta_{c,t}$ . First, we define the mean of  $Y_{ict}$  and  $X_{ict}$  by  $i, c, t$ . We use the "dot" notation for means for brevity. For example,  $\bar{Y}_{ic}$  is the mean of  $Y_{ict}$  averaging over different values of  $t$ .  $\bar{Y}_{i.t}$  is the mean of  $Y_{ict}$  by  $c$ .  $\bar{Y}_{..t}$  is the mean by  $i, c$  and  $t$ . Thus,

$$\begin{aligned} \bar{Y}_{ic} &= \frac{1}{T_{ic}} \sum_{t=1}^{T_{ic}} Y_{ict} \\ \bar{Y}_{i.t} &= \frac{1}{C_{it}} \sum_{c=1}^{C_{it}} Y_{ict} \\ \bar{Y}_{..t} &= \frac{1}{N_{ct}} \sum_{i=1}^{N_{ct}} Y_{ict} \\ \bar{Y}_{i..} &= \frac{1}{C_{it}} \frac{1}{T_{ic}} \sum_{c=1}^{C_{it}} \sum_{t=1}^{T_{ic}} Y_{ict} \\ \bar{Y}_{..t} &= \frac{1}{N_{ct}} \frac{1}{C_{it}} \sum_{i=1}^{N_{ct}} \sum_{c=1}^{C_{it}} Y_{ict} \\ \bar{Y}_{.c.} &= \frac{1}{T_{ic}} \frac{1}{N_{ct}} \sum_{t=1}^{T_{ic}} \sum_{i=1}^{N_{ct}} Y_{ict} \\ \bar{Y}_{...} &= \frac{1}{T_{ic}} \frac{1}{N_{ct}} \frac{1}{C_{it}} \sum_{c=1}^{C_{it}} \sum_{t=1}^{T_{ic}} \sum_{i=1}^{N_{ct}} Y_{ict} \end{aligned}$$

Similarly ,

$$\begin{aligned}
\bar{X}_{ic} &= \frac{1}{T_{ic}} \sum_{t=1}^{T_{ic}} X_{ict} \\
\bar{X}_{i.t} &= \frac{1}{C_{it}} \sum_{c=1}^{C_{it}} X_{ict} \\
\bar{X}_{.ct} &= \frac{1}{N_{ct}} \sum_{i=1}^{N_{ct}} X_{ict} \\
\bar{X}_{i..} &= \frac{1}{C_{it}} \frac{1}{T_{ic}} \sum_{c=1}^{C_{it}} \sum_{t=1}^{T_{ic}} X_{ict} \\
\bar{X}_{..t} &= \frac{1}{N_{ct}} \frac{1}{C_{it}} \sum_{i=1}^{N_{ct}} \sum_{c=1}^{C_{it}} X_{ict} \\
\bar{X}_{.c.} &= \frac{1}{T_{ic}} \frac{1}{N_{ct}} \sum_{t=1}^{T_{ic}} \sum_{i=1}^{N_{ct}} X_{ict} \\
\bar{X}_{...} &= \frac{1}{T_{ic}} \frac{1}{N_{ct}} \frac{1}{C_{it}} \sum_{c=1}^{C_{it}} \sum_{t=1}^{T_{ic}} \sum_{i=1}^{N_{ct}} X_{ict}
\end{aligned}$$

Similar notation applies to  $\delta_{i,c}$ ,  $\delta_{i,t}$  and  $\delta_{c,t}$ .

First, we subtract the average over  $t$ , so that (7) becomes (notice the terms  $\delta_{ic}$  are gone):

$$Y_{ict} - \bar{Y}_{ic} = (X_{ict} - \bar{X}_{ic})' \beta + (\delta_{it} - \bar{\delta}_i) + (\delta_{ct} - \bar{\delta}_c) + (\varepsilon_{ict} - \bar{\varepsilon}_{ic}) \quad (8)$$

Then de-mean (8) over  $c$ , yielding

$$\bar{Y}_{i.t} - \bar{Y}_{i..} = (\bar{X}_{i.t} - \bar{X}_{i..})' \beta + (\delta_{it} - \bar{\delta}_i) + (\bar{\delta}_{.t} - \bar{\delta}_{..}) + (\bar{\varepsilon}_{i.t} - \bar{\varepsilon}_{i..}) \quad (9)$$

Then subtract (9) from (8) (notice  $\delta_{it}$  is gone) :

$$Y_{ict} - \bar{Y}_{ic} - \bar{Y}_{i.t} + \bar{Y}_{i..} = (X_{ict} - \bar{X}_{ic} - \bar{X}_{i.t} + \bar{X}_{i..})' \beta + (\delta_{ct} - \bar{\delta}_c - \bar{\delta}_{.t} + \bar{\delta}_{..}) + (\varepsilon_{ict} - \bar{\varepsilon}_{ic} - \bar{\varepsilon}_{i.t} + \bar{\varepsilon}_{i..}) \quad (10)$$

Now we de-mean (10) over  $i$  :

$$\bar{Y}_{.ct} - \bar{Y}_{.c.} - \bar{Y}_{..t} + \bar{Y}_{...} = (\bar{X}_{.ct} - \bar{X}_{.c.} - \bar{X}_{..t} + \bar{X}_{...})' \beta + (\delta_{ct} - \bar{\delta}_c - \bar{\delta}_{.t} + \bar{\delta}_{..}) + (\varepsilon_{.ct} - \bar{\varepsilon}_{.c.} - \bar{\varepsilon}_{..t} + \bar{\varepsilon}_{...}) \quad (11)$$

Then subtract (11) from (10)(notice  $\delta_{ct}$  is gone):

$$\begin{aligned}
& Y_{ict} - \bar{Y}_{ic.} - \bar{Y}_{i.t} + \bar{Y}_{i..} - \bar{Y}_{.ct} + \bar{Y}_{.c.} + \bar{Y}_{.t} - \bar{Y}_{...} \\
& = (X_{ict} - \bar{X}_{ic.} - \bar{X}_{i.t} + \bar{X}_{i..} - \bar{X}_{.ct} + \bar{X}_{.c.} + \bar{X}_{.t} - \bar{X}_{...})' \theta \\
& + (\varepsilon_{ict} - \bar{\varepsilon}_{ic.} - \bar{\varepsilon}_{i.t} + \bar{\varepsilon}_{i..} - \varepsilon_{.ct} + \bar{\varepsilon}_{.c.} + \bar{\varepsilon}_{.t} - \bar{\varepsilon}_{...})
\end{aligned} \tag{12}$$

Thus, we can rewrite (12) in the following form, and estimate the following equation:

$$\begin{aligned}
& \tilde{Y}_{ict} = \tilde{X}'_{ict} \beta + \tilde{\varepsilon}_{ict} \\
\text{where } & \tilde{Y}_{ict} = Y_{ict} - \bar{Y}_{ic.} - \bar{Y}_{i.t} + \bar{Y}_{i..} - \bar{Y}_{.ct} + \bar{Y}_{.c.} + \bar{Y}_{.t} - \bar{Y}_{...} \\
& \tilde{X}_{ict} = X_{ict} - \bar{X}_{ic.} - \bar{X}_{i.t} + \bar{X}_{i..} - \bar{X}_{.ct} + \bar{X}_{.c.} + \bar{X}_{.t} - \bar{X}_{...} \\
& \tilde{\varepsilon}_{ict} = \varepsilon_{ict} - \bar{\varepsilon}_{ic.} - \bar{\varepsilon}_{i.t} + \bar{\varepsilon}_{i..} - \varepsilon_{.ct} + \bar{\varepsilon}_{.c.} + \bar{\varepsilon}_{.t} - \bar{\varepsilon}_{...}
\end{aligned} \tag{13}$$

We can estimate  $\beta$  using:

$$\hat{\beta} = \left( \tilde{X}'_{ict} \tilde{X}_{ict} \right)^{-1} \tilde{X}_{ict} \tilde{Y}_{ict}$$

and the standard errors using:

$$\begin{aligned}
& \left( \#^{-1} \tilde{X}'_{ict} \tilde{X}_{ict} \right)^{-1} \frac{1}{\sqrt{\#}} \tilde{X}'_{ict} \tilde{\varepsilon}_{ict} \\
& = \left( \#^{-1} \tilde{X}'_{ict} \tilde{X}_{ict} \right)^{-1} \frac{1}{\sqrt{\#}} \sum_{c=1}^{C_{it}} \sum_{t=1}^{T_{ic}} \sum_{i=1}^{N_{ct}} \tilde{X}_{ict} \tilde{\varepsilon}_{ict}
\end{aligned}$$

where  $\#$  is the total number of observations.

In our paper, we estimate the transformed form (13) instead of (7) in the two-stage least square regressions. In the first stage,  $X_{ict}$  is a vector of  $[IV_{c,t} \times X_i \text{ Controls}_{i,c,t}]'$ .  $IV_{c,t}$  include natural disaster shocks, political shocks, revolution shocks and terrorist shocks.  $Y_{ict}$  is the uncertainty measure that we instrumented for.

Then in the second stage, we use the estimated  $\tilde{Y}_{ict}$  from (13) and control variables as a new  $\hat{X}_{ict}$  vector of  $[\tilde{Y}_{ict} \text{ Controls}_{i,c,t}]'$  and  $Y_{ict}$  is the industry growth variable. That is, we estimate the following:

$$Y_{ict} = \delta_{i,c} + \delta_{i,t} + \delta_{c,t} + \hat{X}_{ict} \beta + \varepsilon_{ict}$$

using the demean method again. So that, we can rewrite the estimation equation as

$$\widetilde{Y}_{ict} = \widetilde{X}'_{ict}\beta + \widetilde{\varepsilon}_{ict} \quad (14)$$

$$\begin{aligned} \text{where } \widetilde{Y}_{ict} &= Y_{ict} - \bar{Y}_{ic.} - \bar{Y}_{i.t} + \bar{Y}_{i..} - \bar{Y}_{.ct} + \bar{Y}_{.c.} + \bar{Y}_{.t} - \bar{Y}_{...} \\ \widetilde{X}_{ict} &= \widehat{X}_{ict} - \widehat{X}_{ic.} - \widehat{X}_{i.t} + \widehat{X}_{i..} - \widehat{X}_{.ct} + \widehat{X}_{.c.} + \widehat{X}_{.t} - \widehat{X}_{...} \\ \widetilde{\varepsilon}_{ict} &= \varepsilon_{ict} - \bar{\varepsilon}_{ic.} - \bar{\varepsilon}_{i.t} + \bar{\varepsilon}_{i..} - \varepsilon_{.ct} + \bar{\varepsilon}_{.c.} + \bar{\varepsilon}_{.t} - \bar{\varepsilon}_{...} \end{aligned}$$

We can thus estimate  $\beta$  using:

$$\widehat{\beta} = \left( \widetilde{X}'_{ict}\widetilde{X} \right)^{-1} \widetilde{X}_{ict}\widetilde{Y}_{ict}$$

In general since we do not know the distribution of  $\varepsilon_{ict}$  we do not know the distribution of  $\widetilde{\varepsilon}_{ict}$  either. We test various distributions for  $\widetilde{\varepsilon}_{ict}$ , including bootstrap, clustering and allowing for serially correlated errors. We find that our results are robust to various distributions of  $\widetilde{\varepsilon}_{ict}$ . In the paper, we report results using bootstrapped errors.



Table 7: Country Coverage and Number of Observations

Country	No. of observations	Country	No. of observations
Argentina	961	Kuwait	907
Australia	999	Luxembourg	1,013
Austria	1,013	Mexico	961
Belgium	1,009	Morocco	1,039
Bangladesh	961	Netherlands	1,013
Canada	961	Nigeria	934
China	772	Norway	985
Chile	1,033	New Zealand	1,065
Colombia	1,013	Pakistan	961
Czech Republic	715	Peru	1,065
Denmark	1,013	Philippines	799
Ecuador	1,013	Poland	1,013
Egypt	961	Portugal	1,007
Finland	1,013	Romania	1,039
France	1,013	Russian Federation	499
United Kingdom	1,010	South Africa	1,036
Germany	444	Saudi Arabia	934
Greece	986	Singapore	1,025
Hungary	1,013	Spain	1,011
India	987	Sweden	1,013
Indonesia	1,013	Switzerland	961
Ireland	1,004	Thailand	961
Iran, (Islamic Republic of)	1,013	Tunisia	961
Israel	957	Turkey	961
Italy	1,011	Ukraine	445
Japan	1,013	Venezuela	961
Kenya	1,018	Viet Nam	202
Korea, Republic of	1,039		

## B Basic Data

Table 7 reports an overview of the data by country. Table 8 reports the industry technological characteristics.

Table 8: Industry Technological Measures

Industry	ISIC	EFD	DEP	RND	LAB	FIX	LMP
Food products	311	.0039	7.09	0.073	0.281	0.373	1.195
Beverages	313	.0048	7.09	0.039	0.248	0.372	1.29
Tobacco	314	.0801	5.248	0.222	0.117	0.189	0.815
Textiles	321	0.029	7.665	0.144	0.458	0.345	1.232
Apparel	322	0.075	6.437	0.02	0.447	0.134	1.998
Leather	323	.0959	9.266	0.198	0.444	0.135	1.927
Footwear	324	.045	8.325	0.153	0.446	0.16	2.239
Wood products	331	0.052	9.525	0.032	0.467	0.305	1.72
Furniture, except metal	332	0.015	8.312	0.155	0.488	0.28	1.381
Paper and products	341	.0062	8.632	0.083	0.363	0.472	0.902
Printing and publishing	342	.0222	9.745	0.1	0.407	0.261	1.67
Industrial chemicals	351	0.028	9.646	0.269	0.241	0.381	1.34
Other chemicals	352	1.654	6.888	1.951	0.218	0.207	2.13
Petroleum refineries	353	.0055	6.776	0.057	0.173	0.591	0.763
Misc. pet. and coal products	354	.0059	6.776	0.186	0.3	0.372	1.042
Rubber products	355	.0064	10.072	0.187	0.423	0.322	1.098
Plastic products	356	0.088	10.072	0.171	0.402	0.374	1.557
Pottery, china, earthenware	361	.0107	8.234	0.503	0.475	0.4	1.292
Glass and products	362	0.289	7.554	0.115	0.399	0.4	1.755
Other non.met. Min. prod.	369	0.021	8.234	0.095	0.385	0.48	0.99
Iron and steel	371	.0004	6.578	0.066	0.477	0.427	0.951
Non.ferrous metals	372	0.037	5.393	0.101	0.424	0.364	1.245
Fabricated metal products	381	.0052	7.043	0.147	0.455	0.274	1.365
Machinery, except electrical	382	0.542	8.832	0.933	0.433	0.195	2.694
Machinery, electric	383	0.543	9.381	0.814	0.407	0.208	2.704
Transport equipment	384	0.041	10.559	0.316	0.44	0.264	1.614
Prof. & sci. equip.	385	0.942	9.21	1.194	0.382	0.181	2.79
Other manufactured prod.	390	0.404	10.07	0.302	0.414	0.186	2.006

Note:  $EFD_i$  (external finance dependence),  $DEP_i$  (depreciation),  $RND$  (R&D intensity),  $LAB_i$  (labor intensity),  $FIX_i$  (fixity),  $LMP_i$  (investment lumpiness) are the average of 70s, 80s and 90s from Ilyina and Samaniego (2011); The manufacturing industry classification is 3 digit ISIC rev2.

Table 9: LIML estimation results

This table represents results from the following regression using LIML estimation procedure:

$$Growth_{c,i,t} = \delta_{i,c} + \delta_{i,t} + \delta_{c,t} + \beta(UncertaintyShock_{c,t-1} \times X_i) + \alpha Controls_{i,c,t} + \epsilon_{c,i,t}$$

We only report  $\beta$ . Each cell represents one regression. The dependent variable is industry value added growth rate, output index growth rate and gross output growth rate. Independent variables are the following:  $EFD_i$ (external finance dependence),  $DEP_i$ (depreciation), RND (R&D intensity),  $LAB_i$ (labor intensity),  $FIX_i$ (fixity),  $LMP_i$ (investment lumpiness) are the average of 70s, 80s and 90s from Ilyina and Samaniego (2011). The uncertainty measures are stock market returns, cross section, bond yield and exchange rate volatility from Bloom et al (2012). Standard errors in parentheses, \*\*\* p<0.01, \*\* p<0.05.

$X_i$	Industry growth measure $Growth_{c,i,t}$		
	Value added	Output index	Output
DEP	-.284*** (.0549)	-.007*** (.00235)	-.138** (.0604)
LMP	-.676*** (.194)	-.0239*** (.00695)	-.296*** (.0843)
EFD	-.256* (.133)	-.0137 (.0117)	.0201 (.0586)
RND	-.188* (.110)	-.0247** (.0123)	-.145 (.0983)
LAB	-3.698*** (1.266)	-.0168 (.0324)	-.803 (.544)
FIX	2.164** (.978)	.108*** (.0365)	1.250*** (.483)
Obs	16,149	15,115	16,152

## C Further estimation results

Results using LIML rather than TSLS are reported in the Table 9. Results are generally similar to the basic results reported in the paper. The main difference is that the row for  $FIX_i$  now has significant coefficients for all measures of  $Growth_{c,i,t}$ . However, this is not the case in the TSLS specification.

## D Proofs

First some derivations.

Given  $z$  and  $k$  it is simple to find the optimal value of  $n(z, k) = \left(\frac{\theta z k^\alpha}{w}\right)^{\frac{1}{1-\theta}}$ . Plugging that in we end up with a modified reduced form problem of the form:

$$V(z, k_{-1}; v_t) = \max_k \left\{ \tilde{z} k^{\tilde{\alpha}} - (k - k_{-1}) - \kappa \max\{0, k_{-1} - k\} + \frac{1 - \lambda(z)}{1 + i} EV(z', k(1 - \delta); v') \right\} \quad (15)$$

where  $\tilde{\alpha} = \frac{\alpha}{1-\theta} < 1$  and  $\tilde{z} = \left(\frac{z}{w-\theta}\right)^{\frac{1}{1-\theta}} \left[\theta^{\frac{\theta}{1-\theta}} - \theta^{\frac{1}{1-\theta}}\right]$ .

**Proof of Lemma 1** Consider the transformed problem

$$\begin{aligned} V(z, k_{-1}; v_t) &= \max_{u, h} \left\{ \tilde{z} k^{\tilde{\alpha}} - u - (1 - \kappa) h + \frac{1 - \lambda(z)}{1 + i} EV(z, k(1 - \delta); v_t) \right\} \\ &u \geq 0, h \geq 0 \\ &k = k_{-1} + u - h. \end{aligned}$$

It is straightforward to show that when  $h > 0$  is optimal,  $u = 0$ , since having  $u > 0$  would imply incurring larger costs  $\kappa$  to achieve a desired level of capital  $k$ . Similarly, when  $u > 0$  is optimal,  $h = 0$ , since having  $h > 0$  would imply incurring costs  $\kappa$  to achieve a desired level of capital  $k$  when this is not needed.

**Proof of Proposition 1** The recursive problem has also an infinite-horizon specification that has the same solution. We find it convenient to work with the infinite problem. This is

$$\begin{aligned} &\max \left\{ E \sum_{t=0}^{\infty} \left( \frac{1 - \lambda(z)}{1 + i} \right)^t [\tilde{z}_t k_t^{\tilde{\alpha}} - u_t - (1 - \kappa) h_t] \right\} \\ &u_t \geq 0, h_t \geq 0 \\ &k_t = k_{t-1} (1 - \delta) + u_t - h_t \end{aligned}$$

Writing down the Lagrangian for this problem we obtain:

$$E \sum_{t=0}^{\infty} \left( \frac{1 - \lambda(z)}{1 + i} \right)^t [\tilde{z}_t k_t^{\tilde{\alpha}} - u_t + (1 - \kappa) h_t + \eta_t u_t + \lambda_t h_t + \gamma_t [k_t - k_{t-1} (1 - \delta) - u_t + h_t]]$$

where  $\eta_t$ ,  $\lambda_t$  and  $\gamma_t$  are the multipliers on  $u_t$ ,  $h_t$  and  $k_t$  respectively. The Karush-Kuhn-Tucker

(KKT) conditions for the multipliers are:

$$\eta_t u_t = 0, \lambda_t h_t = 0, \gamma_t [k_t - k_{t-1} (1 - \delta) - u_t + h_t] = 0$$

The derivatives of the Lagrangian yield the following optimality conditions:

$$\tilde{z}_t k_t^{\tilde{\alpha}} - u_t + (1 - \kappa) h_t + \eta_t u_t + \lambda_t h_t + \gamma_t [k_t - k_{t-1} (1 - \delta) - u_t + h_t] + \left( \frac{1 - \lambda(z)}{1 + i} \right) E [\tilde{z}_{t+1} k_{t+1}^{\tilde{\alpha}} - u_{t+1} + (1 - \kappa) h_{t+1}] = 0$$

$$\tilde{\alpha} \tilde{z}_t k_t^{\tilde{\alpha}-1} + \gamma_t - \left( \frac{1 - \lambda(z)}{1 + i} \right) E [\gamma_{t+1} (1 - \delta)] = 0$$

$$-1 + \eta_t - \gamma_t = 0, (1 - \kappa) + \lambda_t + \gamma_t = 0$$

This yields three cases. First, if  $u_t > 0$  (positive investment) then

$$\eta_t = 0, -1 = \gamma_t, \lambda_t = \kappa, h_t = 0$$

and

$$\tilde{\alpha} \tilde{z}_t k_t^{\tilde{\alpha}-1} - 1 = \left( \frac{1 - \lambda(z)}{1 + i} \right) E [\gamma_{t+1} (1 - \delta)] \quad (16)$$

This implies that if firms *invest* they will invest up to the value of  $k_t$  that satisfies this equation. Then, if  $h_t > 0$  (negative investment) then

$$\lambda_t = 0, \gamma_t = -(1 - \kappa), \eta_t = \kappa$$

and

$$\tilde{\alpha} \tilde{z}_t k_t^{\tilde{\alpha}-1} - (1 - \kappa) = \left( \frac{1 - \lambda(z)}{1 + i} \right) E [\gamma_{t+1} (1 - \delta)] \quad (17)$$

This implies that if firms *disinvest* they will do so down to the value of  $k_t$  that satisfies this equation.

If both  $h$  and  $u$  are zero then

$$\eta_t \geq 0, \lambda_t \geq 0, \gamma_t \leq 0$$

and

$$k_t = k_{t-1} (1 - \delta)$$

So that  $\gamma_t$  satisfies:

$$\tilde{\alpha}\tilde{z}_t k_t^{\tilde{\alpha}-1} + \gamma_t - \left( \frac{1 - \lambda(z)}{1 + i} \right) E[\gamma_{t+1} (1 - \delta)] = 0 \quad (18)$$

We also have that

$$\begin{aligned} -1 + \eta_t - \gamma_t &= 0 \\ (1 - \kappa) + \lambda_t + \gamma_t &= 0 \\ \lambda_t + \eta_t &= \kappa \end{aligned}$$

Notice this implies that  $\gamma_t \in [-1, -(1 - \kappa)]$ . The next stage in the proof is to return to the recursive problem and note that the fact that there is a recursive solution implies that  $\gamma_t = \gamma(k_{-1}, z, v)$ ,  $\eta_t = \eta(k_{-1}, z, v)$  and  $\lambda_t = \lambda(k_{-1}, z, v)$ . Our conditions then become

$$\begin{aligned} \eta(k_{-1}, z, v) u(k_{-1}, z, v) &= 0 \\ \lambda(k_{-1}, z, v) h(k_{-1}, z, v) &= 0 \\ \gamma(k_{-1}, z, v) [k(k_{-1}, z, v) - k_{t-1}(1 - \delta) - u(k_{-1}, z, v) + h(k_{-1}, z, v)] &= 0 \\ \tilde{\alpha}\tilde{z}_t k(k_{-1}, z, v)^{\tilde{\alpha}-1} + \gamma(k_{-1}, z, v) - \left( \frac{1 - \lambda(z)}{1 + i} \right) E[\gamma(k, z', v') (1 - \delta)] &= 0 \\ -1 + \eta(k_{-1}, z, v) - \gamma(k_{-1}, z, v) &= 0 \\ (1 - \kappa) + \lambda(k_{-1}, z, v) + \gamma(k_{-1}, z, v) &= 0 \end{aligned}$$

For the case where either of  $h$  or  $u$  are non-zero, this implies that the chosen value of  $k$  when there is investment or disinvestment depends only on  $z$  and  $v$ , since  $k_{-1}$  does not enter the relevant equations (16) and (17). This defines the thresholds  $\bar{k}^*(z, v)$  and  $\underline{k}^*(z, v)$ . In addition, (16) and (17) imply that

$$\bar{k}^*(z, v)^{\tilde{\alpha}-1} = \underline{k}^*(z, v)^{\tilde{\alpha}-1} + \frac{\kappa}{\tilde{\alpha}\tilde{z}_t}$$

which in turn implies that  $\bar{k}^*(z, v) < \underline{k}^*(z, v)$ . Then, for the case where both  $h$  and  $u$  are zero, equation (18) becomes

$$\tilde{\alpha}\tilde{z}_t k_{-1}^{\tilde{\alpha}-1} + \gamma(k_{-1}, z, v) - \left( \frac{1 - \lambda(z)}{1 + i} \right) E[\gamma(k_{-1}(1 - \delta), z', v') (1 - \delta)] = 0. \quad (19)$$

Rearranging this so that  $\gamma(k_{-1}, z, v)$  is a function of other arguments, standard recursive

arguments apply to this problem so that the Bellman operator  $B$  in the following equation is a contraction mapping:

$$B\gamma(k_{-1}, z, v) = \min \left\{ \kappa - 1, \max \left\{ -1, -\tilde{\alpha}\tilde{z}k_{-1}^{\tilde{\alpha}-1} (1 - \delta)^{\tilde{\alpha}-1} + \left( \frac{1 - \delta}{1 + i} \right) (1 - \lambda(z)) \int \int \gamma(k_{-1}(1 - \delta), z', v) \right\} \right\}$$

Assuming that  $\gamma$  is increasing and concave implies that  $B\gamma$  is also increasing and concave (since the sum of concave functions is concave). This completes the proof.

**Proof of Proposition 2** The solution to the decision problem is found by construction in the proof of Proposition 2.1. The fact that, regardless of the original condition, all firms will immediately restrict their capital stocks to being below the supremum of  $\underline{k}^*(z, v)$  means that the measure  $\mu_t$  cannot explode. This is because firms above  $\underline{k}^*(z, v)$  immediately shrink to  $\underline{k}^*(z, v)$ .

**Proof of Proposition 3** Note that all firms either invest to  $\bar{k}^*(z, v)$ , disinvest to  $\underline{k}^*(z, v)$  or depreciate by a factor of  $1 - \delta$ . Thus, for any given value of  $(z, v)$  firms are either at  $\bar{k}^*(z, v)(1 - \delta)^x$  or  $\underline{k}^*(z, v)(1 - \delta)^x$  for some  $x \geq 0$ . Or zero if they are newborn. However since incumbent firms cannot have capital below the lowest value of  $\bar{k}^*(z, v)$  or  $\underline{k}^*(z, v)$  that means that at some point they need to reinvest (or disinvest) back to  $\bar{k}^*(z, v)$  or  $\underline{k}^*(z, v)$ . The exception of course is firms whose initial conditions were not on that grid: however as they depreciate and/or experience shocks at some point they will have to invest or disinvest to  $\bar{k}^*(z, v)$  or  $\underline{k}^*(z, v)$ .