The Embodiment Controversy: on the Policy Implications of Vintage Capital models

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Abstract

We explore the long run impact of policy on the level of economic activity through changes in the vintage distribution of capital, in a model where different vintages coexist in production. Because firms can choose the vintage of capital in which they invest, investment subsidies do not in general affect the vintage structure of capital. In contrast, vintage-specific taxes or subsidies that target the newest vintages of capital can significantly affect output and welfare in the long run, mainly downwards.

Keywords: Embodiment controversy, vintage capital, capital taxation, investment subsidies.

JEL Codes: O11, O13, O16, O41, O47.

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1 Introduction

An extensive literature investigates whether productivity improvements are embodied in capital, in a debate known as the "embodiment controversy." On the one hand, recent developments in theory and in data have increased the popularity of vintage capital models due to their ability to account for US growth and their implications for industry dynamics.[1] At the same time, the *policy implications* of the hallmark of these models – that new vintages of capital are more productive than old vintages – have not been widely explored. This neglect is important: Denison (1964) argues in an oft-quoted comment that the embodiment controversy is unimportant *precisely because it is not policy-relevant*. The argument is that policy would have to induce permanent and unrealistically large changes in investment rates to significantly skew the productivity profile towards newer, more productive vintages of capital.

A key assumption underlying this argument is that all investment must take place in capital of the latest vintage. This assumption is a feature of most vintage capital models, see the survey in Jovanovic and Yatsenko (2012). However, allowing investment in vintages of capital other than the latest is essential for matching the gradual diffusion patterns widely observed in empirical studies on innovations – see Griliches (1957) and Gort and Klepper (1982) among others. If agents may invest not just in the latest vintage of capital but in capital of *earlier* vintages – either through the production of new capital goods of an older vintage, or through purchases or imports of used goods – then aggregate investment rates and the productivity distribution of capital become uncoupled. As a result, if agents choose which vintage of capital to invest in, policy may affect aggregates through the productivity distribution of capital *even if investment rates are held constant.*

This paper explores the policy implications of capital-embodied technical progress in a model where investment is allowed in *any* current or past vintage of capital.

[1] See Johansen (1959), Solow (1960) among others for early contributions, as well as Hercowitz (1998) and Boucekkine et al (2011) for more recent reviews. Greenwood, Hercowitz and Krusell (1997) and Cummins and Violante (2002) argue that models where the productivity of investment improves over time can account for a large share of US economic growth. Samaniego (2010) provides key evidence based on firm dynamics.

Specifically, we study the impact of *vintage-specific* taxes and subsidies, which may distort the agent's decision regarding the choice of vintage. The model is a version of Jovanovic and Yatsenko (2012, henceforth JY12), extended to allow for such transfers. We select this model because it is a suitable workhorse for studying diffusion patterns: it allows different vintages of capital to coexist in the production function through imperfect substitution, and it displays the well-known feature of gradual diffusion curves for new capital goods, as identified in the empirical literature.

The key ingredient of the model, as in Chari and Hopenhayn (1991), is that there is a distinction between the date at which a particular capital good is produced and the *vintage of the technology embodied within.* Consider the example of operating system software, and assume for simplicity that all computers use Windows operating systems. Windows 7 was introduced in 2009, but Microsoft continued to supply Windows XP, and firms could acquire newly produced copies of the older operating system for some years.[2] Moreover, firms that did purchase Windows 7 might do so without necessarily replacing their computer hardware of an older vintage. The reason why the ability to invest in capital of an older vintage is important for the policy implications of vintage capital is that, as Denison (1964) observes, if all investment is only in the newest vintage then the only channel through which policy can impact aggregates is through changes in net investment rates. In this example, this would amount to forcing firms which buy a new operating system to buy Windows 7. In contrast, if investment does not necessarily have to be in the newest vintage, then expenditure on operating systems is no longer tied to expenditure on the *newest* operating systems. As a result, even without changes in investment rates, there could be a significant impact of policy on output and welfare in a vintage capital world if policy can skew the *vintage composition of investment*.[3] Of course investment rates

[2] The software example is also useful because Windows XP was subject to several free updates and improvements gradually over time, a feature captured in the model and interpreted as a component of "learning". It is this "learning" which leads to the gradual adoption of newer vintages. The presumption in a vintage capital model would be then that Windows 7 is more productive than Windows XP, conditional on similar updates and learning.

[3] Intuitively, if the technology for producing capital increases everywhere at a rate γ, but in one country policy induces investment to occur on average in vintages of capital that are s years older than in another, yet the investment rate is the same, then GDP would be $s \times \gamma$ lower in the first

may also be affected by policy, so a contribution of the paper will be a *quantitative* assessment of the impact of policy on aggregates in general equilibrium, as well as an assessment that abstracts from changes in investment rates.

The policies we examine are vintage-specific taxes and subsidies. We show analytically that the vintage distribution is insensitive to transfers that are *not* vintage-specific, so that blanket investment subsidies have no impact on aggregates through the vintage distribution. Then, we analyze the impact of policies that differentially subsidize (or tax) the newest year of capital vintages. We analyze the *long-run* impact of such policies on allocations, via their impact on the stationary equilibrium. In order to focus on the impact of changes in the vintage distribution on aggregates we analyze two types of inter-vintage transfer schemes: (1) where there are no net subsidies to capital, and separately (2) where transfers to capital are such that there is no impact on aggregate investment.[4] The structure of the exercise is similar to that in Restuccia and Rogerson (2008), who assess the impact of policies that result in intra-firm resource reallocation by means of firm-specific transfer schemes. In contrast, our focus is on the vintage distribution.

We find that subsidies to new vintages financed out of taxing older vintages are detrimental to welfare and to GDP in the long-run. In the calibrated economy, a 20 percent subsidy to investment in the newest vintage leads to a decline in consumption in each period of 1 percent, a 50 percent subsidy lowers consumption by 5 percent, and a 100 percent subsidy lowers consumption by fully 18 percent. Moreover, this impact is not due to any inherent waste in the tax system: these results are for transfer schemes such that there are no net transfers to or from the capital goods sector. We obtain similar results when the transfer scheme is designed to ensure that there is no impact of aggregate investment: thus, these effects are entirely due to distortions in the vintage composition of investment.[5] The conclusion is that

country than in the second at all dates. Since in principle s is unbounded, factors that affect the average vintage of capital used could lead countries to differ in terms of income by a significant amount.

[4]It turns out that results are similar, because the net transfers required to keep aggregate investment constant are small.

[5]It is well known since at least Diamond and Mirlees (1971) that production subsidies are inefficient absent other market failures. The point is that, in principle, distortions that affect

policy-induced distortions to the vintage distribution can have significant impact on welfare. Since this impact is potentially large, the paper identifies an as yet unexplored channel whereby policy, financing frictions or other distortions might lead to differences in macroeconomic outcomes among developed and developing economies – a channel that can only be studied in a model where technical progress is at least partly embodied in capital.

Section 2 discusses the literature. Section 3 describes the economic environment and solves for equilibrium. Section 4 calibrates the model and reports the results of quantitative policy experiments. Section 5 concludes with suggestions for future work.

2 Motivation and Literature

Denison (1964) argues that the existence (or not) of capital embodied technical progress is not important for policymakers, because unreasonably large changes in the age structure of capital would be necessary for policy to significantly influence aggregates. Much of the related literature has focused on assessing whether productivity improvements in capital are an important factor of growth (e.g. Hulten (1992), or Greenwood et al (1997)), without addressing this key criticism: that an important factor in evaluating the usefulness of vintage capital models is the assessment of the *policy-relevance of changes in the vintage distribution of capital.*

Assessing the impact of policy on the vintage distribution requires a model which accounts for basic properties of the vintage distribution of capital. First, different vintages must coexist in production. Second, the model should reproduce basic features of the vintage distribution – in particular, the slow diffusion of new capital goods. Third, as a result, the model should allow investment to occur not just in the latest vintage of capital, but in older vintages too. This feature requires a distinction between the age of a capital good and its *technological* vintage. For example, while technological progress implies that the most powerful computer available improves

specifically the vintage distribution can have significant aggregate impact. Blanket subsidies to investment may affect aggregates but, as we show, they have no impact on the vintage distribution.

over time, computers of lesser power continue to be produced, using other than the latest processors. A consequence is that, even if investment rates do not change over time or are unresponsive to the policy environment, the technological vintage of the capital created via investment could be responsive to policy. The productivity distribution of capital could change significantly in terms of *technological* vintage, even in the absence of changes in investment rates.

Most vintage capital models are inadequate for performing this assessment. The reason is that most models either assume that all investment occur in the newest vintage of capital, or they assume that there is a choice of vintage but the optimal choice is always the newest.[6] Instead, this paper adopts the framework recently introduced in JY12. In this framework, as in Chari and Hopenhayn (1991), investment may in principle occur in *any* current or past vintage of capital. The reason agent find it optimal to do so is that different vintages are imperfect substitutes in production. The gradual diffusion of new capital goods is achieved via the introduction of vintage-specific learning, which accumulates gradually over time. The model is simple and easily mapped into the data typically used in calibrating models in the related literature – a feature that will be important for generating quantitative results.

Although the main gap in the literature we address is theoretical and quantitative – whether in principle distortions to the vintage distribution can impact economic aggregates – it is worth asking whether there are any real-life policies that are more directed at some vintages than others. In practice in the United States investment tax credits are formulated to promote particular new technologies,[7] e.g. the 2009 Car Allowance Rebate System (known more widely as "cash-for-clunkers"), or the Production Tax Credit adopted in 1992 and the more recent Investment Tax Credit, which are directed at the promotion of new wind and solar energy investments respectively. All of these are recent technologies, whereas older systems for transportation or power generation would not benefit from these policies. In addition, Eaton and

[6] See the survey in JY12.

[7] In general the formulation of the Investment Credit (IRS Form 3468) is targeted towards investments of recent vintage.

Kortum (2001) argue that most countries in fact import much of their capital stock from a few advanced economies. In those cases, there exists a way for developing economies to differentially tax new and old capital: by treating new and used capital differently when it is imported. For example, if it takes a year for capital to significantly enter the used capital market, then differential tariffs on imports of new or old capital are equivalent to vintage-specific taxes.[8]

Before specifying the details of the model, we ask: is there any evidence that there do exist differences in the vintage distribution of capital around the world? This is hard to determine if we take seriously the distinction between the age of capital and the vintage of the technology used to make it. However, the motor vehicle industry stands out as one where this distinction may not be so critical. Motor vehicles are often produced with a vintage attached to them, and in all countries the existence of an active secondary market for motor vehicles implies that there is a choice between new and old vehicles, including to some extent imported used vehicles.[9] There is of course significant heterogeneity among vehicles of similar vintage: for example, the quality differences between a 2017 Toyota Corolla and a 2017 Ferrari F12 are not just related to their vintage, and this heterogeneity could hamper inference about the productivity of capital based solely on measured vintages.[10] However, this should be less the case for vehicles used in *public transport*. This is what we focus on, using data provided by the United Nations Economic Commission for Europe (UNECE). The advantage of using European data is that the existence of open markets implies that the second hand market among the countries in the data is relatively fluid, so there is easy access to vehicles of older vintage in these countries. Figure 1 shows that the age distribution is generally tipped towards older vintages of public transport

[8]In practice used machinery tends to experience higher trade barriers than new machinery, including outright prohibition, see United States Department of Commerce (2015) for a global survey. Some authors such as Soloaga et al (1999) argue that in developed economies the opposite could be desirable.

[9]Most countries impose some limits on the ability to import used vehicles, although these tend to be weaker for commercial vehicles, see US Department of Commerce (2015).

[10]This need not be a problem in principle, since the difference we are interested are between a 2017 Toyota Corolla and a 2007 Toyota Corolla, but in practice we want to know that differences in age distributions are likely due to difference in vintage composition, not to other sources of quality difference.

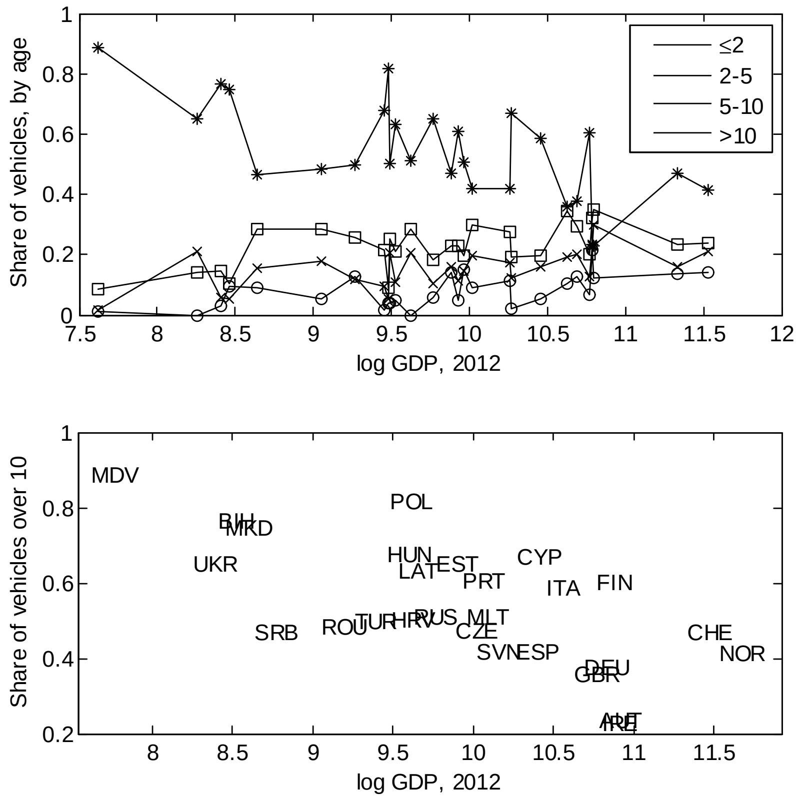

Figure 1 – Share of Motor Coaches, Buses and Trolleybuses at end 2012, by age, in selected countries indicated by ISO codes. The correlation in the lower panel is -0.67. Sources – UNECE, World Bank, own calculations.

vehicles in lower income countries, where income is measured using GDP per capita (PPP adjusted) in 2012, as reported by the World Bank. The relationship is strong: a 100 percent increase in GDP per capita is associated with a fully 11 percentage point decrease in the share of public transport motor vehicles older than 10 years. We do not infer from this anything about the particular policies that might either lead to these outcomes or that might be used to overcome them, although this would be interesting to study: the observation is simply that vintage distributions do appear to vary around the world.

In the remainder of the paper we focus on inter-vintage policy experiments in the calibrated economy.

3 Economic Environment

We extend the framework of JY12 to allow for inter-vintage transfer schemes. Consider a continuous time market economy with a population of unit mass. Each agent is endowed with a unit flow of labor each date t which they may supply to the labor market. Utility is defined over streams of consumption:

$$U = \int_0^\infty \frac{c_t^{1-\eta} - 1}{1-\eta} dt,\ c : \mathbb{R}^+ \to \mathbb{R}^+. \tag{1}$$

There is a production technology that produces a final good y_t, which can be used for consumption or for investment. The production function is:

$$y_t = K_t^\alpha N_t^{1-\alpha},\ \alpha \in (0,1) \tag{2}$$

where N_t is labor used at date t and K_t is aggregate capital, defined below.

Each date t a new investment technology becomes available, referred to as a *vintage*. Agents may invest u_{vt} units of the final good in producing capital of any vintage $v \leq t$, and there is a stock of capital of any vintage k_{vt}. Aggregate capital is defined as

$$K_t = \left[\int_{-\infty}^t A_{t-v} \left(z_v k_{vt} \right)^\beta dv \right]^{\frac{1}{\beta}} \tag{3}$$

where z_v is a productivity level embodied in capital of vintage v, and A_{t-v} is a *learning function* associated with capital of age $t - v$. Parameter β is related to the elasticity of substitution among vintages: if σ is the elasticity of substitution between vintages, then $\beta \equiv \frac{\sigma - 1}{\sigma}$, or $\sigma = \frac{1}{1-\beta}$. The purpose of the learning function (see JY12) is to capture the empirical fact that new products (including capital goods) tend to diffuse slowly, so the peak in use of new goods is not until several years after their introduction, e.g. see Gort and Klepper (1982). If A_{t-v} were a constant, and if $z_v > z_{v-1}\ \forall v$ (the hallmark of a vintage capital model) then we would have that $k_{vt} > k_{v-1,t}\ \forall v, t$.

In what remains of the paper we will assume that A_s is an increasing function ($A_s > A_{s-\Delta}$ for any $\Delta > 0$), and that $z_v = e^{\gamma v}$.

Notice that learning depends on the age of capital. There are alternative specifications of learning-by-doing technologies, where the learning occurs depending on past use e.g. Jovanovic and Lach (1989). However the goal of the paper is to establish that the vintage distribution is sensitive to policy, so the exact form of the learning function is secondary.[11] The distinction in our context will only be important *quantitatively* if learning functions are affected significantly by vintage-specific transfer schemes. However, since the learning about a technology is something that occurs through worldwide use, the actual pattern of the learning function is likely exogenous to any particular firm or country, especially if the country is small or not producing a lot of R&D specifically in the field of application of that technology, which Eaton and Kortum (2001) argue is the empirically relevant case for most of the world. In any case it would be interesting in future work to explore the impact of different determinants of learning.

The stock of physical capital of any particular vintage k_{vt} accumulates according to:

$$\frac{\partial k_{vt}}{\partial t} = u_{vt} - \delta k_{vt} \tag{4}$$

where δ is the depreciation rate and u_{vt} is investment in capital of vintage v. Thus, the feasibility condition for the economy is

$$y_t \geq c_t + \int_{-\infty}^{t} u_{vt} dv. \tag{5}$$

At date 0, the quantity k_{v0} is given for all $v \leq 0$ at date zero. It is then straight-

[11] Samaniego (2006c) uses this learning function to account for the productivity slowdown of the 1970s, based on the advent of a new type of capital (information technology) for which prior learning is not fully applicable. Thus, vintage differences in learning can be interpreted as technological *switching costs*.

forward to show that (4) implies that

$$k_{vt} = e^{-\delta(t-v)}x_v + \int_v^t e^{-\delta(t-s)}u_{v,s}ds,\ v > 0 \tag{6}$$

$$k_{vt} = e^{-\delta(t-v)}k_{v0} + \int_0^t e^{-\delta(t-s)}u_{v,s}ds,\ v \leq 0 \tag{7}$$

where x_v is investment in new capital at the moment it was new.

At each date t firms solve:

$$\max_{KN_t} \{y_t - r_t K_t - w_t N_t\} \tag{8}$$

subject to the production function.

Example 1 *Before closing the model we can use the production technology to ask: what is the difference in the productivity of 2 economies with a different vintage structure? Imposing the fact that $N_t = 1$ in equilibrium, we have that output is given by:*

$$y_t = \left[\int_{-\infty}^t A_{t-v}\left(e^{\gamma v}k_{vt}\right)^\beta dv\right]^{\frac{\alpha}{\beta}} \tag{9}$$

Consider any continuous k_{vt}. If there is another economy with a distribution $\tilde{k}_{vt}$ such that $\int_{-\infty}^t k_{vt}dv = \int_{-\infty}^t \tilde{k}_{vt}dv$, where $\tilde{k}_{vt}$ first-order stochastically dominates k_{vt} in terms of the age of capital, then the economy with $\tilde{k}_{vt}$ will have higher output $\tilde{y}_t$ than the other, even though the total physical units of capital are the same in both economies. Indeed, if the dominance is sufficiently significant, there is unbounded impact that the vintage distribution, even if the savings rate in both economies is the same. To see this, suppose the distributions $k_{v,t}$ and $\tilde{k}_{v+\bar{s},t}$ are given, such that $k_{v+\bar{s},t}$ is a downward translation of the distribution $\tilde{k}_{v+\bar{s},t}$:

$$k_{v,t} = \begin{cases} 0 & t - v < \bar{s} \\ \tilde{k}_{v+\bar{s},t} & t - v \geq \bar{s} \end{cases}$$

Here the distribution of $k_{v.t}$ is the same as $\tilde{k}_{v+\bar{s},t}$, so the entire distribution is shifted down by $\bar{s}$. Again, the two have identical mass in terms of raw units of capital.

However, it is straightforward to show that

$$\frac{y_t}{\tilde{y}_t} = e^{-\alpha\gamma\bar{s}}$$

so that, as $\bar{s} \to \infty$, $\frac{y_t}{\tilde{y}_t} \to 0$. *The example shows that distortions to the vintage distribution can have arbitrarily large impact on aggregates even with constant savings rates.*

3.1 Vintage-specific transfers

Assume there is a tax $\tau(t-v) - 1$ on investment of age $t-v$, u_{vt}. Thus instead of paying 1 for a unit of investment, they pay $\tau(t-v)$, where $\tau : \mathbb{R}^+ \to \mathbb{R}^+$ is twice continuously differentiable. Thus τ is a multiplicative price wedge on investment. The revenues are distributed lump sum to consumers T_t, leading to the budget condition:

$$T_t = \int_{-\infty}^{t} [\tau(t-v) - 1] \, u_{vt} dv \tag{10}$$

If τ is increasing then newer vintages of capital are favored by the tax system, through lower taxes, tax rebates, or subsidies. The agent's budget constraint is then

$$c_t + \int_{-\infty}^{t} u_{vt} dv \leq r_t K_t + w_t N_t \tag{11}$$

3.2 Model Solution

Definition 1 *An equilibrium of the model is a set of prices such that, given the initial condition* k_{v0} $(v \leq 0)$, *the agent chooses investment* u_{vt} *and consumption* c_t *at each date so as to maximize* (1) *subject to* (3) *and* (11), *firms maximize* (8) *and the government satisfies* (10) *for all* t.

Definition 2 *A stationary equilibrium is an equilibrium and an initial condition* k_{v0} $(v \leq 0)$ *such that the age distribution of capital is constant over time and the growth rate of consumption* g *is constant over time.*

We now study some properties of the equilibrium and of the stationary equilibrium. All proofs are in the Appendix.

Proposition 1 *There exists a unique stationary equilibrium.*

First of all, in a model without taxes ($\tau = 1$), the user cost of capital of any vintage v is equivalent to $r + \delta$, and investment will be chosen optimally so as to equalize this and the marginal product of capital of each vintage v. However, with a non-trivial tax scheme, the user cost of capital and therefore the marginal product is affected by the tax scheme in two ways. First, the level of $\tau(v)$ affects the level of the user cost of capital. Second, marginal variation in the tax scheme by vintage $\tau'(v)$ also affects the optimal investment rate in each vintage. For example, if there is a range of v over which the tax rate $\tau(v)$ is flat and then rises rapidly, the marginal cost of new capital will be constant and then rise. As the acceleration begins, agents find it preferable to invest in the vintage with the low tax rather than the otherwise very similar vintage with the higher tax rate (or lower subsidy rate). Given the tax scheme, this investment profile is optimal.

Proposition 2 *When investment is optimal,*

$$\alpha z_v^{\beta} y_t^{\frac{\alpha-\beta}{\alpha}} A_{t-v} k_{vt}^{\beta-1} = (r+\delta)\,\tau(t-v) - \tau'(t-v). \tag{12}$$

Proposition 3 $k_{vt} = e^{gt}\chi_{t-v}$*, where* χ_{t-v} *depends neither on* t *nor* v *independently.*

Proposition 4 $u_{vt} = e^{gt}\kappa_{t-v}$*, where* κ_{t-v} *depends neither on* t *nor* v *independently.*

A consequence of these results is that capital and investment of any given age s are a constant share of GDP over time, even if the share of any particular vintage rises and then falls over time.

Definition 3 *The age distribution at date* t *is defined as the density function:*

$$\hat{k}_{st} \equiv \frac{k_{st}}{\int_0^\infty k_{ut}du} \frac{\chi_{st}}{\int_0^\infty \chi_{ut}du} \tag{13}$$

Having defined this density, we can make two observations about the model economy.

Proposition 5 *A vintage-independent tax or subsidy $\tau(s) = \bar{\tau}$ does not affect the age distribution.*

Proposition 5 has important implications. When the choice of technological vintage is distinct from the date of production, investment taxes (or subsidies) cannot be justified as policies that stimulate investment in new technology. Since investment can occur in capital goods produced using a variety of capital producing technologies, both new and not-so-new, a tax on investment in itself does nothing to skew the vintage structure of capital.

Furthermore, there is a sense in which the overall amount of taxes and subsidies towards or away from capital does not affect the age distribution of capital, the subject of this paper. Rather, only the *relative sensitivity* of transfers to the vintage does. Consider that any vintage-specific transfer scheme $\tau(s)$ can be formulated as a profile $\tau(s) \equiv \bar{\tau}\eta(s)$, where $\eta(\cdot)$ is a relative sensitivity to vintage and $\bar{\tau}$ is a constant related to revenue generation.

Corollary 1 *Considering that any vintage-specific transfer scheme $\tau(s)$ can be formulated as $\tau(s) \equiv \bar{\tau}\eta(s)$, the value of the constant $\bar{\tau}$ does not affect the age distribution.*

For learning and taxation profiles that can be interpreted in terms of rates of sensitivity to age, we can deliver a further result about the age distribution of capital.

To do so, we consider a special case. Assume that $A_s = e^{\theta s}$. In this case $\theta > 0$ is a parameter that can be interpreted as a Poisson rate at which agents learn about different vintages of technology. Furthermore, assume that $\tau(s) = \bar{\tau}e^{\omega s}$. Parameter $\bar{\tau}$ captures the overall size of the transfer scheme, and parameter $\omega \lesseqgtr 0$ captures the rate at which the transfer scheme favors capital of different vintages, so that higher ω implies relatively higher taxation of old capital (and relative higher subsidization of new capital). Under this parameterization, taxing new capital relatively less than the old (i.e., higher ω) can be shown analytically to skew the age distribution of capital towards newer vintages.

Proposition 6 *Assume that $A_s = e^{\theta s}$ ($\theta > 0$). Consider economies $i \in \{1, 2\}$, such that $\tau(s) = \bar{\tau} e^{\omega_i s}$, and assume that $\beta\gamma - \theta + \omega_i (1 - \beta) > 0 \; \forall i$. The age distribution of capital $\hat{k}_s$ in economy 1 first-order stochastically dominates that in 2 iff $\omega_1 < \omega_2$.*

Remark 1 *When there is no taxation, Jovanovic and Yatsenko (2012) show that when $A_s = e^{\theta s}$, if economy 1 has higher β, higher γ or lower θ than economy 2, then the vintage distribution in 1 first order stochastically dominates that in 2. The same holds true in our environment with taxation when $\tau(s) = \bar{\tau} e^{\omega_i s}$. However, since the current paper is concerned with the impact of policy $\tau(\cdot)$ on the vintage distribution, we keep constant technological parameters such as β, γ and θ in our thought experiments and numerical experiments.*

4 Quantitative evaluation

4.1 Calibration

We calibrate the model economy in order to perform quantitative policy experiments. Although the model is formulated in continuous time, we need a unit for measuring time in order to calibrate parameters in a consistent manner. We measure time in years. Details of the computational procedure are in the Appendix.

We require functional forms for the tax function $\tau(\cdot)$ and for the learning function $A(\cdot)$. For the calibration process we set $\tau(s) = 1$, so there are no inter-vintage transfers, and calibrate the model to US data, which is relatively unregulated and for which the related literature reports a wealth of relevant data. Later we discuss the inter-vintage transfer schemes we consider for the policy experiments.

We set $A_s = 1 - e^{-\phi s}$ where $\phi > 0$. In this way, learning about any particular vintage is bounded, so sooner or later all vintages become for obsolete for any $\gamma > 0$. This implies that investment in the newest vintage u_{tt} equals zero, consistent with the observation that new capital diffuses gradually rather than exhibiting "jumps". Later we assess the sensitivity of results to this assumption.

Given these choices of functional form, the parameters to be calibrated are γ, α, δ, σ, ρ, η and ϕ.

We set $\alpha = 0.33$, a standard value for the capital share of income. This is consistent with the idea that the learning is not embodied in the physical capital itself but in some other resource – for example, in the labor that uses the capital, or in the productivity of the firm that uses the capital as in Samaniego (2010).[12] However, for robustness later we allow for larger values of α, which is equivalent to interpreting "capital" as including other accumulable resources that might embody the learning. Note that assuming a small value of α is a conservative assumption in the sense that it limits the impact of changes in the vintage distribution on aggregates. If $\alpha = 0$, then capital and the vintage distribution are irrelevant for aggregate outcomes).

Since $g = \frac{\alpha}{1-\alpha}\gamma$, allowing for 1.5 percent annual GDP growth as is typically found in US data would imply that $\gamma = 0.0350$. However this number is very elevated compared to empirical estimates. The reason is that such an approach to calibration assumes that all growth is due to capital-embodied growth, as in Solow (1960). If instead we view γ as reflecting improvements in the marginal rate of transformation between consumption and new capital goods (including quality improvements to capital) as in Greenwood et al (1997) and Cummins and Violante (2002) among others, then we can match γ using the growth rate of the quality-adjusted relative price of capital. Using the values from Greenwood et al (1997) we have that[13] $\gamma = 0.018$, so that $g = 0.0077$. The remainder of growth is due to unexplained technical progress that is outside the model.[14]

An important parameter is the elasticity of substitution among vintages σ. We set $\sigma = 2$. JY12 argue that $\sigma \approx 2$ based on the estimates of Bahk and Gort (1993) for

[12] Profits that accrue to entrepreneurs who use their labor to create firms is not capital income, see Gollin (2002).

[13] This is the average rate across equipment and structures used in that paper.

[14] If overall growth is 1.5 percent as in JY12, this value of γ accounts for 51 percent or about half of growth. If overall growth is 1.24 percent as in Greenwood et al (1997) then γ accounts for about 60 percent of growth, as they find.

Table 1: Calibration Statistics

Calibration parameters for the benchmark economy. Calibration assumes there are no inter-vintage transfers.

Parameter	Interpretation	Value
γ	Rate of capital embodied tech. prog.	0.018
α	Capital share of income	0.33
δ	Physical depreciation rate	0.06
σ	Cross-vintage substitution elasticity	2
ρ	Discount rate	0.01
η	Intertemporal elasticity of substitution	1
ϕ	Speed of learning	0.6

just 2 types of capital, new and old. Independently, Edgerton (2011) finds estimates based on looking at the substitutability between new and old capital, ranging from 1.7 to 10.5 depending on the type of capital, with the estimates clustered towards the lower range. This implies that $\beta = 0.5$. We also examine the impact of larger values of σ.

Another key model parameter is the speed of learning ϕ. JY12 set $\phi = 0.6$ based on the finding of Bahk and Gort (1993) that most vintage-specific learning appears to be complete after 6 years.[15]

Finally, we set $\delta = 0.06$, $\rho = .01$ and $\eta = 1$, all of which are standard values in a growth accounting context. See Table 1 for all parameter values.

The calibrated model displays reasonable investment behavior. First, in the calibrated economy, we find that the investment share of GDP is 18.5 percent. This is very close to the value in US data, even though this parameter was not directly calibrated. Also, Figure 2 shows that the diffusion pattern is an S-shape followed by a gentle decline, as found by Gort and Klepper (1982) for a variety of capital goods. This is due to the initially gradual adoption of each vintage of capital due to learning, followed by a gentle decline as investment shifts towards newer vintages and the older capital depreciates. The peak in usage is when the capital is about 7 years of age – although the peak in *investment* is much earlier, between the first

[15] The value $\phi = 0.6$ stems from assuming that exactly 95 percent of the learning is complete by the 6^{th} year.

and second year of introduction. This reflects the finding of Bahk and Gort (1993) that vintage-specific learning is in general quite rapid, along with the fact that, in a relative sense, the learning is counteracted by the advance in the productivity of newly introduced vintages.

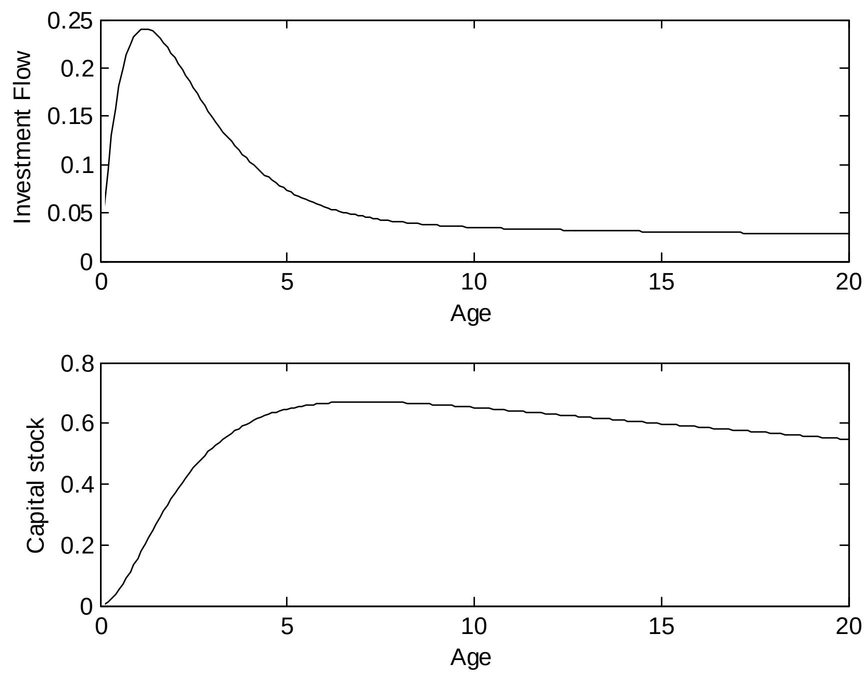

Figure 2 – Investment $u_{v,t}$ and capital stock $k_{v,t}$ based on age $t - v$ in the calibrated model economy, in model units.

4.2 Policy experiments

In the remainder of the paper we focus on inter-vintage policy experiments in the calibrated economy.

For these experiments, we must choose a specific inter-vintage transfer scheme $\tau(\cdot)$. We examine the impact of transfers either too or from capital of the "newest vintage", interpreted as capital produced using technology introduced during the most recent *year*.

We choose this transfer scheme for the following reasons. First, Denison's (1956) criticism conceives of the policy implications of vintage capital models in this fashion, shifting resources towards the technology of the latest vintage. Second, it is not unusual in policy circles to discuss investment tax credits (i.e. subsidies) as being useful for targeting new technologies. As shown by Proposition 5, an investment tax credit or subsidy that is not vintage-specific will not affect the vintage distribution, as firms could write off the tax credit for investment in new capital of *any* vintage. However, in practice in the United States investment tax credits are formulated to promote particular new technologies, as discussed in Section 2. In addition, differential treatment of capital imports depending on whether they are new or used is equivalent to a tax directed at older vintages.

We examine two types of tax schemes:

- Schemes with no net transfers to or from capital;
- Schemes where net transfers to or from capital are enough to keep aggregate investment constant.

In our benchmark results, we look at tax schemes such that there are *no net transfers to or from capital*, i.e. $T_t = 0$. The reason we focus on policies with no net transfers to capital is in order to focus on strictly *inter-vintage* transfers: the results using any policy that allows $T_t \neq 0$ would conflate the impact of policy through the vintage distribution with its redistributive impact.[16]

We do so as follows. Let τ_0 be the tax rate for firms below one year of age. Let $\tau_0 + \tau_{diff}$ be the tax rate for firms above one year of age. Then let τ_{diff} reflects (in levels) the preferential tax treatment given to newer vintages. The government budget balance condition would then become

$$[\tau_0 - 1] \int_0^1 u_{vt} dv + [\tau_{diff} + \tau_0 - 1] \int_1^\infty u_{vt} dv = 0. \tag{14}$$

[16] The review in Samaniego (2006a) finds that, at least among OECD countries, there are no net transfers to or from firms.

Given a value of τ_{diff}, we can raise or lower τ_0 so as to ensure that is condition is met.

Two important technical notes are in order regarding this transfer scheme.

1. As specified, the tax scheme is not continuously differentiable, whereas to solve the model we require it to be at least twice continuously differentiable, because the second derivative of $\tau(\cdot)$ enters the optimal decision rule for investment u_{vt}. As a result, we use a smooth approximation to the above "jumping" transfer scheme. In practice we use the following:

$$\tau(s) = \tau_0 + \tau_{diff}\Phi(s|1,\varsigma) \tag{15}$$

   where $\Phi(s|1,\varsigma)$ is the cumulative distribution function of the normal distribution with mean one and standard deviation ς. The balanced budget condition 14 must be modified accordingly:

$$\int_0^\infty [\tau_0 + \tau_{diff}\Phi(s|1,\varsigma) - 1]\, u_{vt} dv = 0 \tag{16}$$

   The key to ensuring (14) and (16) are similar is to set ς to a small value, so that the transition between tax rates τ_0 and $\tau_0 + \tau_{diff}$ is rapid. We set $\varsigma = 0.001$, which implies that capital of vintage two days less than a year is taxed at a rate negligibly different from τ_0, and that capital of vintage two days more than a year is taxed at a rate negligibly different from $\tau_0 + \tau_{diff}$.

2. For a given value of $\tau_{diff} \neq 0$, it is not necessarily the case that there exists a value of τ_0 that satisfies the balanced budget condition (16). For example, if the relative subsidy τ_{diff} is very large, the subsidy on young capital may be so much that it cannot be financed only through taxing old capital – of which there may be little, especially if σ is high so capital of different vintages are very good substitutes. As vintages become perfect substitutes ($\sigma \to \infty$) then small tax differentials between different vintages will result in huge differences in investment patterns, so that practically all investment is directed towards the subsidized vintages, so that government budget balance is not possible for

sufficiently large values of τ_{diff}. Still, we are interested in schemes that do satisfy these properties are of interest because they allow us to understand the impact of distortions to the vintage distribution in a controlled environment.

What is the impact of such a tax scheme on diffusion patterns? Proposition 2 indicates that optimal investment u_{vt} is affected by the structure of the tax system. With the tax system defined by equation (15), when investment tax rates jump up or jump down around age 1, investment patterns may change suddenly. When $\tau_{diff} > 0$ (so new vintages are taxed less) Figure 3 shows that investment drops off in general for vintages older than 1. Close to 1, there is a spike as the tax rate accelerates from τ_0 towards $\tau_0 + \tau_{diff}$, as it is more profitable to invest in those vintages than in other vintages that are similar technologically but very different for tax purposes. This is followed by a sharp drop as the tax rate slows down and approaches $\tau_0 + \tau_{diff}$. In contrast, when $\tau_{diff} < 0$ (so new vintages are taxed more), Figure 3 shows that investment *rises* in general for vintages older than 1. Close to 1, there is a sharp drop as the tax rate declines from τ_0 towards $\tau_0 + \tau_{diff}$, followed by a sharp rise as the tax rate settles down and approaches $\tau_0 + \tau_{diff}$.

As mentioned, separately, we also examine policies that are designed to keep investment constant. In these experiments given a value of τ_{diff}, we select τ_0 so that investment equals 18.5 percent of GDP as in the benchmark economy. In this case we will have that

$$\int_0^\infty \left[\tau_0 + \tau_{diff}\Phi\left(s|1,\varsigma\right) - 1\right] u_{vt} dv = T_t,\, T_t \gtrless 0. \tag{17}$$

The point of this experiment is to demonstrate that results for the other type of policy are not primarily due to the aggregate impact of changes in investment rates: they are due to distortions in the vintage structure. Indeed results turn out to be very similar as the required values of T_t are small. Again, it is not the case that for any particular value of τ_{diff} it is possible to find a transfer scheme that satisfies these properties.

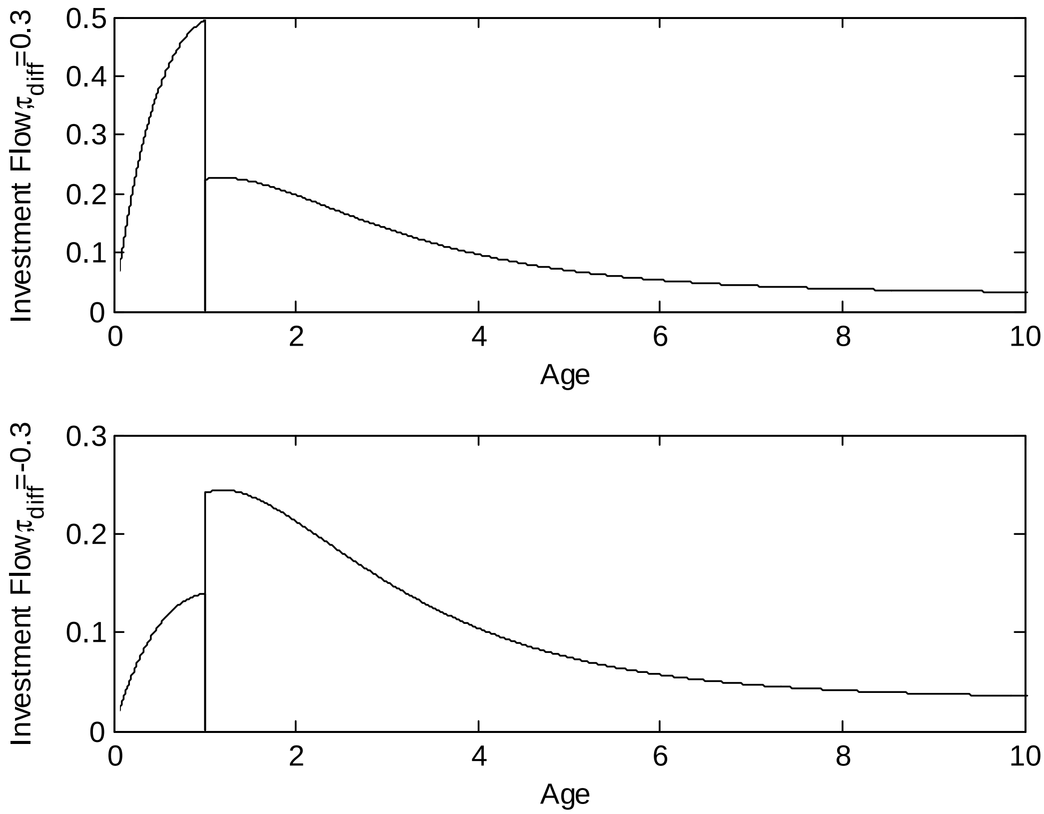

Figure 3 – Investment u_{vt} by age when capital with v under 1 is taxed differentially from capital with v above 1. In the top panel old capital is taxed more $(\tau_{diff} = 0.3)$. In the lower panel new capital is taxed more $(\tau_{diff} = -0.3)$.

4.3 Results

Before anything, it is not necessarily the case that the steady state impact of taxes and transfers is negative – even though the usual welfare theorems apply to the model economy. The reason is simple: we are comparing across steady state economies, which have different initial values of $k_{v,0}$, $v < 0$. The welfare theorems apply to the model economy with a *given* initial condition $k_{v,0}$. Nonetheless, as we shall see, there does not appear to be much scope for increasing long run welfare through intra-vintage transfers. In general it is not clear whether subsidizing the new is going to increase or decrease long-run welfare, since new capital is less productive in the sense of learning but more productive in terms of γ, and learning is rapid. We measure welfare changes using the percentage change (relative to the calibrated benchmark) in the level of consumption in each period, similar to a dynamic compensating variation.

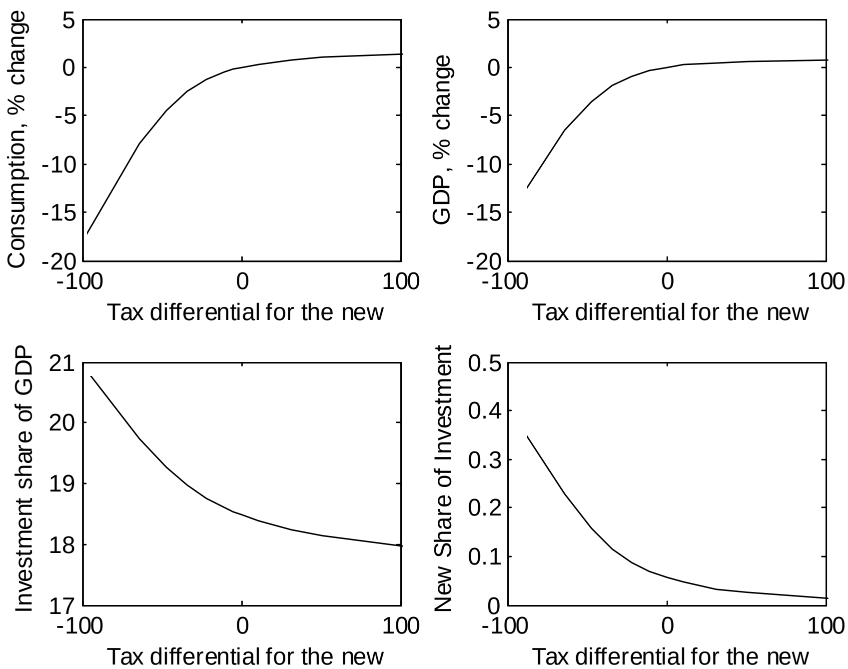

Figure 4 – Impact of inter-vintage transfer systems. The x-axis in each case is the percent tax premium on capital of vintage over one year, τ_{diff}. The figure and all figures below assume that there are no net transfers to capital unless otherwise indicated.

Figure 4 shows that subsidizing the new (a negative tax differential) actually *decreases* welfare in the calibrated economy. A 20 percent subsidy on investment in the newest vintages (conditional on overall transfers to capital being zero), leads to a decline in consumption in each period of 1 percent. The impact of such transfers is non-linear: a 50 percent subsidy lowers consumption by 5 percent, and a 100 percent subsidy lowers consumption by fully 18 percent. In contrast, a 50 percent *tax* on the new (which is equivalent to a small subsidy to older capital) increases consumption in each period by about 1 percent. These long-run gains increase with greater taxation of the new, peaking around 1.5 percent when $\tau_{diff} = 300$ percent (not shown in Figure 4) and then fading gradually: at this point, there is very little investment in new capital because of the onerous taxation.

Interestingly, Figure 4 shows that the impact of vintage-specific taxation on *overall* investment is not significant. Varying τ_{diff} between -100 percent and $+100$ percent decreases investment from about 21 percent of GDP down to about 18 percent (the baseline value is 18.5 percent). On the other hand, the share of investment devoted to investment of the newest vintage (again, defined as the newest *year* of vintages) varies significantly, from about 40 percent down to almost zero (compared to the baseline value of 5.7 percent). This suggests that it is the distortions to the vintage structure – not changes in aggregate investment – that are responsible for the results.

This is confirmed in Figure 5. Figure 5 reports results for transfer systems where τ_0 and therefore T_t are chosen so as to keep aggregate investment constant. The results concerning welfare as measured by detrended consumption, as well as GDP and the share of investment in new vintages, are very similar. In addition, varying τ_{diff} between -100 and $+100$ percent only entails net transfers to consumers from the capital sector of $+2$ to -0.5 percent.

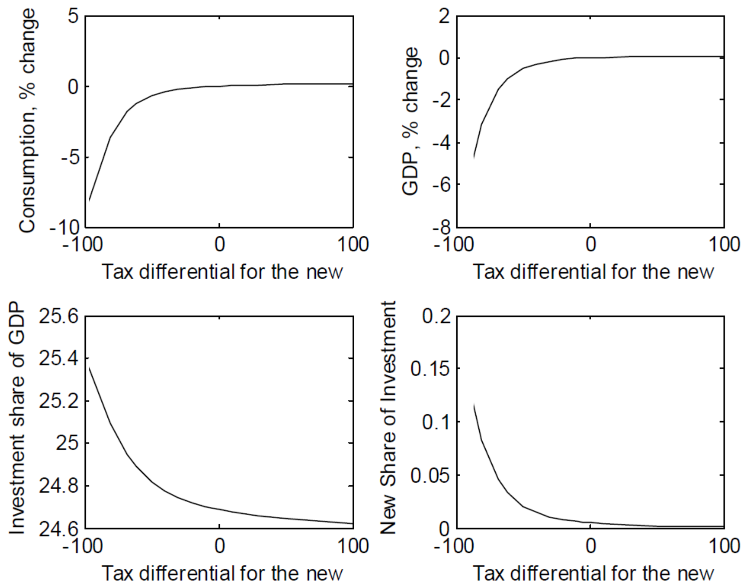

Figure 5 – Impact of inter-vintage transfer systems. The x-axis in each case is the percent tax premium on capital of vintage over one year, τ_{diff}. This Figure assumes that transfers are set so that investment is constant.

4.4 Robustness: the impact of learning

One might ask whether the negative impact of new vintage subsidies in Figure 4 is because of the assumption that initial productivity of new capital is zero. To examine this question, we modify the learning function so that:

$$A_s = 1 - e^{-\phi(s+\bar{s})}, \bar{s} \geq 0.$$

Allowing the parameter $\bar{s} > 0$ is equivalent to assuming that investment in any vintage of capital jumps from zero to a positive value when $v = t$. We set $\bar{s} = 1.4$, which is about the age that maximizes the investment flow in the baseline calibration. Figure 6 shows that allowing $\bar{s} > 0$ can actually *increase* the macroeconomic impact of vintage-specific transfers, although the difference is not very large compared to the baseline with $\bar{s} = 0$. The upside remains small in the long run (around 2% of consumption) but the downside can be even larger than before. This suggests that the shape of the learning profile, while important for matching diffusion curves, is not critical for the policy implications of inter-vintage transfers: instead, the productivity differences between vintages, and the difficulty of substituting between different vintages, are important.

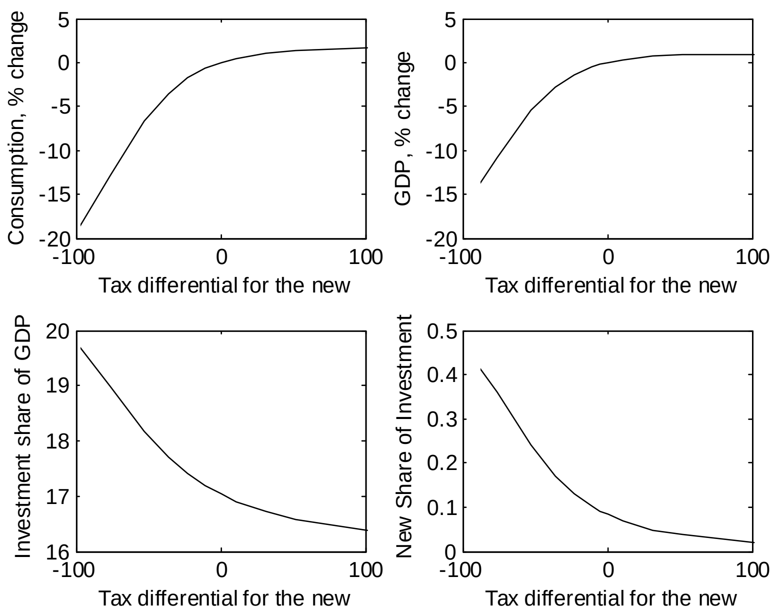

Figure 6 – Impact of inter-vintage transfer systems. The x-axis in each case is the percent tax premium on capital of vintage over one year, τ_{diff}. Assumes initial learning A_0 is positive: $A_s = 1 - e^{-\phi(s+\bar{s})}$ and $\bar{s} \geq 0$.

4.5 Robustness: the impact of embodiment

In the model there are two reasons why inter-vintage transfers might have aggregate impact. One is the fact that the vintages have different productivity. The other is that they are simply imperfect substitutes. To see whether embodiment (rather than substitution alone) is important we perform two exercises. First, in Figure 7 we repeat the experiments with a low value of γ. Second, in Figure 8 we raise the elasticity of substitution σ to a larger value.

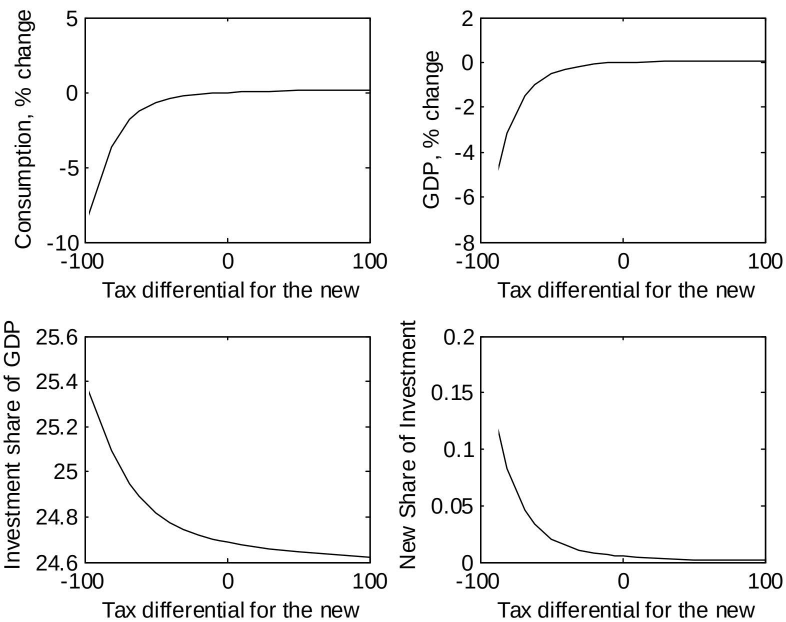

Figure 7 – Impact of inter-vintage transfer systems. The x-axis in each case is the percent tax premium on capital of vintage over one year, τ_{diff}. Assumes γ equals 0.001.

Figure 7 distinguishes between the impact of embodiment and the impact of substitution among vintages by assuming γ is small. When $\gamma = 0.001$, compared to the calibrated value of $\gamma = 0.018$, the impact of taxation on long-run consumption declines by more than half. For example, whereas a 100 percent subsidy to new vintages lowers consumption by 18 percent, when γ is small consumption declines by only about 8 percent. Thus it is not just the fact that old and new capital are not perfect substitutes that affects the results: the rate of capital embodied technical progress is a key determinant of the results.

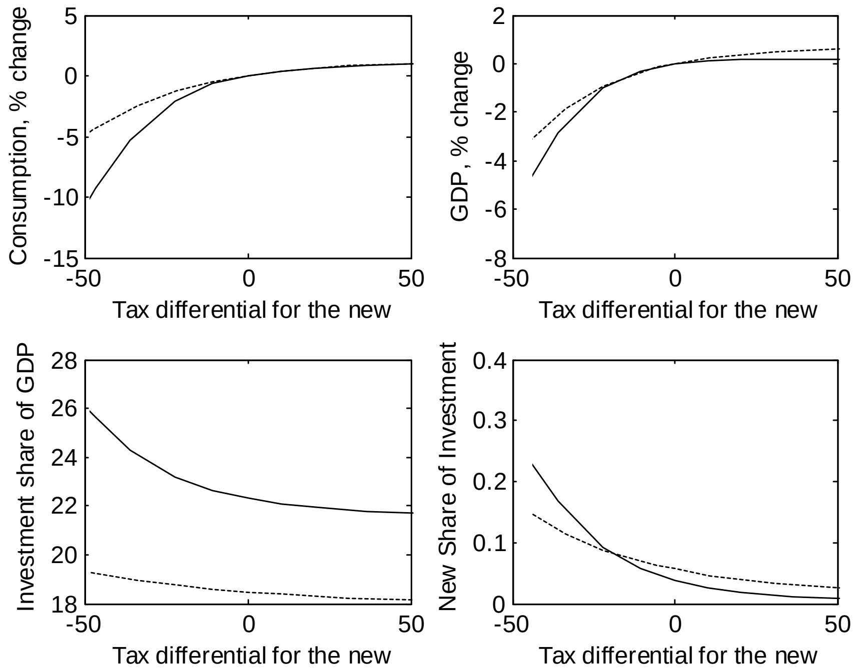

Figure 8 Impact of inter-vintage transfer systems. The x-axis in each case is the percent tax premium on capital of vintage over one year, τ_{diff}. The thick line assumes σ equals 4 and the dotted line assumes that σ equals 2 as in the benchmark calibration. Tax differentials τ_{diff} smaller than minus 50 percent do not satisfy the balanced budget condition (16) when σ equals 4 so they are not displayed.

Next, Figure 8 shows the results for $\sigma = 4$, to examine the sensitivity of results to the inter-vintage elasticity of substitution. When $\tau_{diff} = 50\%$, consumption drops relative to the untaxed economy by about 11 percent. In contrast, in the baseline scenario with $\sigma = 2$ a tax differential $\tau_{diff} = 50\%$ leads consumption to drop relative to the untaxed economy by about 5 percent. When $\sigma = 4$, different vintages are more substitutable, so a given tax differential leads to more drastic differences in investment patterns, which then interact more strongly with vintage productivity differences. In this sense, our benchmark assumption that $\sigma = 2$ is conservative.

Finally we also check the sensitivity of results to interpreting capital K_t as including not just physical capital but also whatever resource embodies the learning A_s. See Figure 9, where we assume that $\bar{s} = 0$ as in the baseline economy, but raise the capital share to $\alpha = 0.5$. In this case the impact of inter-vintage transfers is larger, as might be expected, since capital (and hence distortions to capital) are more important for output and consumption when α is large. Whereas $\tau_{diff} = -100$ percent led to a decline in long-run consumption of 18 percent in the baseline economy with $\alpha = 0.33$, when $\alpha = 0.5$ consumption declines by 34 percent.

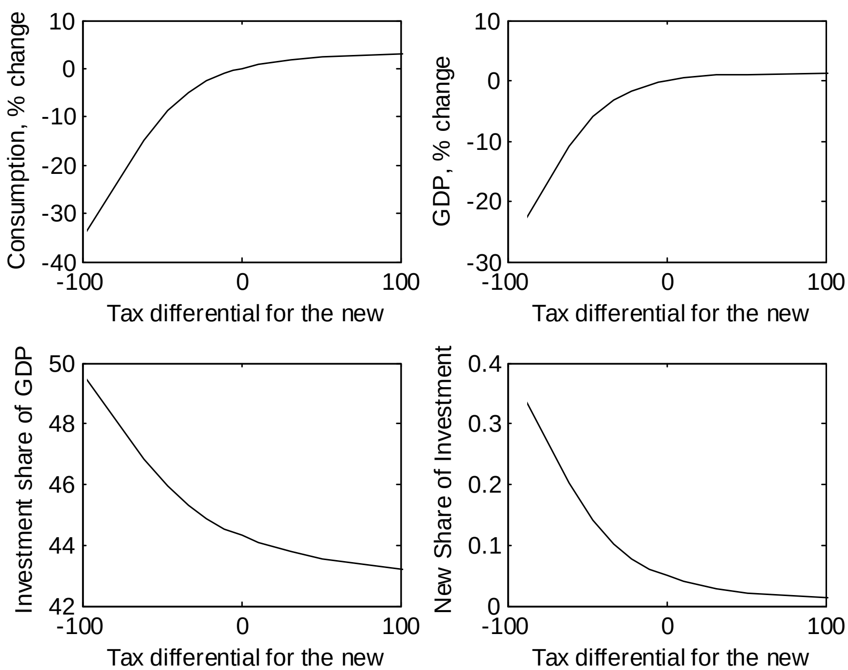

Figure 9 – Impact of inter-vintage transfer systems. The x-axis in each case is the percent tax premium on capital of vintage over one year, τ_{diff}. Assumes α equals 0.5.

5 Concluding remarks

The paper finds that policy-based distortions to the vintage distribution of capital can have significant aggregate and welfare impact. The results provide a clear response to a central argument in the embodiment controversy that the usefulness of models where technical progress is embodied in capital hinges on their policy-relevance. They also indicate a new channel for policy, market frictions or other distortions to affect the wealth of nations – a channel that can only be addressed using a model where technical progress is embodied in capital.

The paper strictly focuses on the impact of policy through the vintage distribution. We do not mean to suggest that there are not channels other than the vintage distribution through which policy might affect aggregates which relate to environments where technology is embodied in capital. One possibility is the fact, documented in Cummins and Violante (2002), that differences in rates of technology improvement vary across capital goods. Thus, changes in the *composition* of capital – not the *vintage* distribution, but the *type* distribution – could matter too. There could also be interactions between regulation and vintage capital through *firm dynamics*, as suggested by Samaniego (2006a, 2006b, 2010), which could be propagated through the choice of vintage. These questions remains for future work.

Another channel from which we abstract is a potential interaction with vintage-specific *human* capital, or with the skill composition of the economy, as suggested by Chari and Hopenhayn (1991). Extending the model to allow vintage physical and vintage human capital accumulation to interact would likely amplify the results of the paper.

We do not study the distinction between used and new capital of a given vintage. This distinction could matter in an environment where there is a concept of reallocation among production units, and where there might be costs of reallocation. Lanteri (2016) studies such reallocation but in an environment without a vintage model. Eisfeldt and Rampini (2007) find that used capital is important for the operation of credit constrained firms, suggesting that changes in the vintage distribution could be important for the aggregate impact of financing frictions.Also, given the large potential impact of distortions to the vintage distribution identified in this paper, it may be important to evaluate whether the fact that imports of used capital goods are restricted or prohibited in developing economies is offset by the costs of ensuring that quality used goods of older vintage might be smoothly imported.

Finally, the model implies that vintage-specific taxation could influence capital prices or investment patterns. For example, trading turnover in certain capital goods is non-monotonic in vintage, as shown by Stolyarov (2002). It would be interesting to explore whether the tax treatment of goods of different vintages, for example differences between the tax treatment of depreciation and actual physical or economic depreciation patterns, could be responsible for non-monotonicity in resale or pricing patterns of used or of old-vintage capital.

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A Proofs

Below are proofs for the results reported in the main text.

Proof of Proposition 1. The proof is a consequence of the results below, which construct a unique stationary equilibrium for the model economy. ■

Proof of Proposition 2. In equilibrium $N_t = 1$ since labor does not enter the utility function, and $w_t = (1-\alpha)\, y_t$. Thus, the Lagrangian for the investment problem is

$$L = \int_0^\infty e^{-rt} \left\{ K_t^\alpha - x_t \tau(0) - \int_{-\infty}^t \tau(t-v)\, u_{vt} dv + \right. \tag{18}$$

$$+ \int_0^t \lambda_{vt} \left[e^{-\delta(t-v)} x_v + \int_v^t e^{-\delta(t-s)} u_{v,s} ds - k_{vt} \right] dv \tag{19}$$

$$\left. + \int_{-\infty}^0 \lambda_{vt} \left[e^{-\delta(t-v)} k_{v0} + \int_0^t e^{-\delta(t-s)} u_{v,s} ds - k_{vt} \right] dv \right\} dt \tag{20}$$

Using small variations of the controls $\delta u, \delta x, \delta y, \delta k$, we have

$$\delta L \approx \int_0^\infty e^{-rt} \left\{ \delta y_t - \delta x_t \tau(0) - \int_{-\infty}^t \tau(t-v)\, \delta u_{vt} dv \right. \tag{21}$$

$$+ \int_0^t \lambda_{vt} \left[e^{-\delta(t-v)} \delta x_v + \int_v^t e^{-\delta(t-s)} \delta u_{v,s} ds - \delta k_{vt} \right] dv \tag{22}$$

$$\left. + \int_{-\infty}^0 \lambda_{vt} \left[\int_0^t e^{-\delta(t-s)} \delta u_{v,s} ds - \delta k_{vt} \right] dv \right\} dt \tag{23}$$

where since $y_t = \left[\int_{v \in \{-\infty, t\}} A_{t-v} (z_v k_{vt})^\beta dv \right]^{\frac{\alpha}{\beta}}$ we have that:

$$\delta y_t = \alpha \left[\int_{v \in \{-\infty, t\}} A_{t-v} (z_v k_{vt})^\beta dv \right]^{\frac{\alpha-\beta}{\beta}} \int_{-\infty}^t A_{t-v} z_v^\beta k_{vt}^{\beta-1} \delta k_{vt} dv$$

Then

$$\delta L \approx \int_0^\infty e^{-rt}\left\{\delta y_t - \delta x_t \tau(0) - \int_{-\infty}^t \tau(t-v)\,\delta u_{vt} dv\right\} dt \tag{24}$$

$$+ \int_0^\infty e^{-rt} \int_0^t \lambda_{vt}\left[e^{-\delta(t-v)}\delta x_v + \int_v^t e^{-\delta(t-s)}\delta u_{v,s} ds - \delta k_{vt}\right] dvdt \tag{25}$$

$$+ \int_0^\infty e^{-rt} \int_{-\infty}^0 \lambda_{vt}\left[\int_0^t e^{-\delta(t-s)}\delta u_{v,s} ds - \delta k_{vt}\right] dvdt \tag{26}$$

then

$$\delta L \approx \int_0^\infty \int_0^s \lambda_{ts} e^{-rs-\delta(s-t)}\delta x_t dtds - \int_0^\infty e^{-rt}\left\{\delta x_t \tau(0)\right\} dt \tag{27}$$

$$+ \int_0^\infty e^{-rt}\left\{-\int_{-\infty}^t \tau(t-v)\,\delta u_{vt} dv\right\} dt \tag{28}$$

$$+ \int_0^\infty e^{-rt} \int_{-\infty}^t \lambda_{vt}\left[\int_v^t e^{-\delta(t-s)}\delta u_{v,s} ds\right] dvdt \tag{29}$$

$$+ \int_0^\infty e^{-rt}\left\{\alpha y_t^{\frac{\alpha-\beta}{\alpha}} \int_{-\infty}^t A_{t-v} z_v^\beta k_{vt}^{\beta-1}\delta k_{vt} dv\right\} dt \tag{30}$$

$$+ \int_0^\infty e^{-rt} \int_{-\infty}^t \lambda_{vt}\left[-\delta k_{vt}\right] dvdt \tag{31}$$

Switching the integrals and rearranging (see Hritonenko and Yatsenko (1996, 2005)),

$$\delta L \approx \int_0^\infty \left[\int_t^\infty e^{-rs-\delta(s-t)}\lambda_{ts} ds - e^{-rt}\tau(0)\right]\delta x_t dt \tag{32}$$

$$+ \int_0^\infty \int_{-\infty}^t \left[\int_t^\infty e^{-rs-\delta(s-t)}\lambda_{vs} ds - e^{-rt}\tau(t-v)\right]\delta u_{vt} dvdt \tag{33}$$

$$+ \int_0^\infty \int_{-\infty}^t e^{-rt}\left\{\alpha y_t^{\frac{\alpha-\beta}{\alpha}} A_{t-v} z_v^\beta k_{vt}^{\beta-1}\delta - \lambda_{vt}\right\}\delta k_{vt} dvdt \tag{34}$$

Setting the coeffs of δ to zero yields the following optimality conditions:

$$\int_t^\infty e^{-rs-\delta(s-t)}\lambda_{ts}ds = e^{-rt}\tau(0), \forall t \tag{35}$$

$$\int_t^\infty e^{-rs-\delta(s-t)}\lambda_{vs}ds = e^{-rt}\tau(t-v), \forall t, v \tag{36}$$

$$\alpha y_t^{\frac{\alpha-\beta}{\alpha}} A_{t-v} z_v^\beta k_{vt}^{\beta-1} = \lambda_{vt}, \forall t, v \tag{37}$$

Thus, optimally investment is chosen so that

$$\alpha z_v^\beta \int_t^\infty e^{-(r+\delta)s} y_s^{\frac{\alpha-\beta}{\alpha}} A_{s-v} k_{vs}^{\beta-1} ds = e^{-(r+\delta)t}\tau(t-v)$$

Notice this is the same as the solution for JY12 except that $\widehat{z_v^\beta} = z_v^\beta/\tau(t-v)$. Differentiating this condition wrt t yields

$$\alpha z_v^\beta y_t^{\frac{\alpha-\beta}{\alpha}} A_{t-v} k_{vt}^{\beta-1} = (r+\delta)\tau(t-v) - \tau'(t-v).$$

where $\tau'(s)$ is the derivative of the tax system, and $\tau'(0) = \lim_{s\to 0^+}\tau'(s)$. ■

Proof of Propositions 3 **and** 4. We use some proportionality relationships regarding how aggregate variables must grow in a stationary equilibrium, in order to derive further results regarding optimal investment. Recall that:

$$z_v = e^{\gamma v}$$

The optimal growth rate of consumption given the utility function (1) is:

$$g = \frac{r-\rho}{\eta}$$

The fraction of investment in GDP is constant, and K_t grows at the rate $\gamma + g$. This implies as in typical models with capital-embodied technical progress that:

$$g = \frac{\alpha}{1-\alpha}\gamma$$

Now consider that $y_t^{\frac{\alpha-\beta}{\alpha}} \propto e^{gt\frac{\alpha-\beta}{\alpha}}$, and $z_v^\beta \propto e^{\beta\gamma v} = e^{\beta\frac{1-\alpha}{\alpha}gv}$. Since $gt\frac{\alpha-\beta}{\alpha} + \beta\frac{1-\alpha}{\alpha}gv =$

$(1-\beta)\, gt - \beta\gamma\,(t-v)$ we have that (conjecture

$$\begin{aligned} k_{vt} &= \left(\frac{\alpha z_v^\beta y_t^{\frac{\alpha-\beta}{\alpha}} A_{t-v}}{(r+\delta)\,\tau\,(t-v) - \tau'\,(t-v)} \right)^{\frac{1}{1-\beta}} \\ &= \left(\frac{\alpha z_0^\beta y_0^{\frac{\alpha-\beta}{\alpha}} e^{(1-\beta)gt-\beta\gamma(t-v)} A_{t-v}}{(r+\delta)\,\tau\,(t-v) - \tau'\,(t-v)} \right)^{\frac{1}{1-\beta}} \\ &= e^{gt}\chi_{t-v} \end{aligned}$$

where

$$\begin{aligned} \chi_s &= \left(\frac{\alpha z_0^\beta y_0^{\frac{\alpha-\beta}{\alpha}} e^{-\beta\gamma s} A_s}{(r+\delta)\,\tau\,(s) - \tau'\,(s)} \right)^{\frac{1}{1-\beta}} \\ &= \bar{k} \left(\frac{e^{-\beta\gamma s} A_s}{\tau\,(s) - \frac{\tau'(s)}{(r+\delta)}} \right)^{\frac{1}{1-\beta}} \end{aligned}$$

and

$$\bar{k} = \left((r+\delta)^{-1}\, \alpha z_0^\beta y_0^{\frac{\alpha-\beta}{\alpha}} \right)^{\frac{1}{1-\beta}},\ z_0 = 1$$

Since

$$y_t = \left[\int_{-\infty}^{t} A_{t-v}\,(z_v k_{vt})^\beta\, dv \right]^{\frac{1}{\beta}\alpha} \tag{38}$$

this becomes

$$\bar{k} = \left((r+\delta)^{-1}\, \alpha y_0^{\frac{\alpha-\beta}{\alpha}} \right)^{\frac{1}{1-\beta}} = \left((r+\delta)^{-1}\, \alpha \left[\int_{-\infty}^{0} A_{-v} k_{v0}^\beta dv \right]^{\frac{\alpha-\beta}{\beta}} \right)^{\frac{1}{1-\beta}}$$

and then

$$\bar{k} = \left((r+\delta)^{-1} \alpha \left[\int_{-\infty}^{0} A_{-v} \left[e^{g0} \chi_{0-v} \right]^{\beta} dv \right]^{\frac{\alpha-\beta}{\beta}} \right)^{\frac{1}{1-\beta}}$$

$$= \left((r+\delta)^{-1} \alpha \left[\int_{-\infty}^{0} A_{-v} \left[\bar{k} \left(\frac{e^{\beta\gamma v} A_{-v}}{\tau(-v) - \frac{\tau'(-v)}{r+\delta}} \right)^{\frac{1}{1-\beta}} \right]^{\beta} dv \right]^{\frac{\alpha-\beta}{\beta}} \right)^{\frac{1}{1-\beta}}$$

Rearranging we end up with:

$$\bar{k} = \left[(r+\delta)^{-1} \alpha \right]^{\frac{1}{1-\alpha}} \left[\int_{0}^{\infty} A_s \left[\left(\frac{e^{-\beta\gamma s} A_s}{\tau(s) - \frac{\tau'(s)}{r+\delta}} \right)^{\frac{1}{1-\beta}} \right]^{\beta} ds \right]^{\frac{\alpha-\beta}{\beta(1-\alpha)}}$$

Next we turn to the calculation of optimal investment. For *new capital*,

$$x_t = k_{tt} = e^{gt}\chi_0 = e^{gt}\bar{k} \left(\frac{A_0}{\tau(0) - \frac{\tau'(0)}{(r+\delta)}} \right)^{\frac{1}{1-\beta}}$$

which will be a constant fraction of GDP. For *old capital*, the capital accumulation equation (4) and the above derivations imply that

$$u_{vt} = g e^{gt} \chi_{t-v} + e^{gt} \frac{\partial \chi_{t-v}}{\partial t} + \delta k_{vt} = e^{gt} \left[(g+\delta)\chi_{t-v} + \frac{\partial \chi_{t-v}}{\partial t} \right]$$

Since

$$\frac{\partial \chi_{t-v}}{\partial t} = \bar{k} \frac{\left[e^{-\frac{\beta\gamma(t-v)}{1-\beta}} \frac{dA_{t-v}^{\frac{1}{1-\beta}}}{dt} \right]}{\left[\tau(t-v) - \frac{\tau'(t-v)}{(r+\delta)} \right]^{\frac{1}{1-\beta}}} - \frac{\beta\gamma}{1-\beta}\chi_{t-v}$$
$$-\bar{k} \frac{\frac{1}{1-\beta} e^{-\frac{\beta\gamma(t-v)}{1-\beta}} A_{t-v}^{\frac{1}{1-\beta}} \left[\tau(t-v) - \frac{\tau'(t-v)}{(r+\delta)} \right]^{\frac{1}{1-\beta}-1} \left[\tau'(t-v) - \frac{\tau''(t-v)}{(r+\delta)} \right]}{\left[\tau(t-v) - \frac{\tau'(t-v)}{(r+\delta)} \right]^{\frac{2}{1-\beta}}}$$

we have that

$$\frac{\partial \chi_{t-v}}{\partial t} = \bar{k}\left[e^{-\frac{\beta\gamma(t-v)}{1-\beta}}\frac{dA_{t-v}^{\frac{1}{1-\beta}}}{dt} - \frac{\beta\gamma}{1-\beta}e^{-\frac{\beta\gamma(t-v)}{1-\beta}}A_{t-v}^{\frac{1}{1-\beta}}\right]\left[\tau(t-v) - \frac{\tau'(t-v)}{(r+\delta)}\right]^{\frac{-1}{1-\beta}}$$

$$-\left(\frac{1}{1-\beta}\right)\bar{k}e^{-\frac{\beta\gamma(t-v)}{1-\beta}}A_{t-v}^{\frac{1}{1-\beta}}\left[\tau(t-v) - \frac{\tau'(t-v)}{(r+\delta)}\right]^{\frac{-1}{1-\beta}-1}\left[\tau'(t-v) - \frac{\tau''(t-v)}{(r+\delta)}\right]$$

So $u_{vt} = e^{gt}\kappa_{t-v}$ where

$$\kappa_{t-v} = \frac{\bar{k}e^{-\frac{\beta\gamma(t-v)}{1-\beta}}}{\left[\tau(t-v) - \frac{\tau'(t-v)}{(r+\delta)}\right]^{\frac{1}{1-\beta}}}\left[(g+\delta)A_{t-v}^{\frac{1}{1-\beta}} + \left[\frac{dA_{t-v}^{\frac{1}{1-\beta}}}{dt} - \frac{\beta\gamma}{1-\beta}A_{t-v}^{\frac{1}{1-\beta}}\right]\right.$$

$$\left. -\left(\frac{1}{1-\beta}\right)A_{t-v}^{\frac{1}{1-\beta}}\frac{\left[\tau'(t-v) - \frac{\tau''(t-v)}{(r+\delta)}\right]}{\left[\tau(t-v) - \frac{\tau'(t-v)}{(r+\delta)}\right]}\right].$$

■

Proof of Proposition 5. When $\tau(s) = \bar{\tau}$, the tax and its derivative cancel out of both the numerator and denominator of the vintage distribution in equation (13):

$$\hat{k}_s = \frac{\frac{e^{-\frac{\beta}{1-\beta}\gamma s}A_s^{\frac{1}{1-\beta}}}{\tau(s)-\frac{\tau'(s)}{(r+\delta)}}}{\int_0^\infty\left[\frac{e^{-\frac{\beta}{1-\beta}\gamma u}A_u^{\frac{1}{1-\beta}}}{\tau(u)-\frac{\tau'(u)}{(r+\delta)}}\right]du} = \frac{\frac{e^{-\frac{\beta}{1-\beta}\gamma s}A_s^{\frac{1}{1-\beta}}}{\bar{\tau}}}{\int_0^\infty\left[\frac{e^{-\frac{\beta}{1-\beta}\gamma u}A_u^{\frac{1}{1-\beta}}}{\bar{\tau}}\right]du}. \tag{39}$$

■

Proof of Corollary 1. It is straightforward to show that

$$\hat{k}_s = \frac{e^{-\frac{\beta}{1-\beta}\gamma s}A_s^{\frac{1}{1-\beta}}}{\eta(s) - \frac{\eta'(s)}{(r+\delta)}}\left[\int_0^\infty\left[\frac{e^{-\frac{\beta}{1-\beta}\gamma u}A_u^{\frac{1}{1-\beta}}}{\eta(u) - \frac{\eta'(u)}{(r+\delta)}}\right]du\right]^{-1},$$

so the value of $\bar{\tau}$ is irrelevant for $\hat{k}_s$. ■

Proof of Proposition 6. The distribution of capital vintages is

$$\hat{k}_s = \frac{e^{-\frac{\beta}{1-\beta}\gamma s} e^{\frac{\theta s}{1-\beta}} e^{-\omega s}}{\int_0^\infty \left[e^{-\frac{\beta}{1-\beta}\gamma u} e^{\frac{\theta u}{1-\beta}} e^{-\omega u} \right] du}$$

Then assuming $\beta\gamma - \theta + \omega(1-\beta) > 0$ this reduces to

$$\hat{k}_s = \left[\frac{\beta}{1-\beta}\gamma - \frac{\theta}{1-\beta} + \omega \right] e^{-\left[\frac{\beta}{1-\beta}\gamma - \frac{\theta}{1-\beta} + \omega\right] s}.$$

■

B Computational procedure

We compute the model economy with and without taxation by means of quadrature approximation. Functions such as u_{vt} and k_{vt} are defined continuously. Then each integral required to compute K_t or y_t is evaluated using quadrature approximation by evaluating these functions at small time intervals up to some date T. The date $T = 1000$ years was chosen so that vintages $v \geq T$ would be negligible in the production function. The time interval was chosen to be small. Results are reported using 100 time intervals per year: using 1000 time intervals per year did not change the results at a precision of 6 significant figures.